



From αβγ to udscbt

Contemporary ideas on the structure of things

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outline





u Up	c Charm	t Top
d Down	s Strange	b Bottom

Contemporary ideas on the structure of things

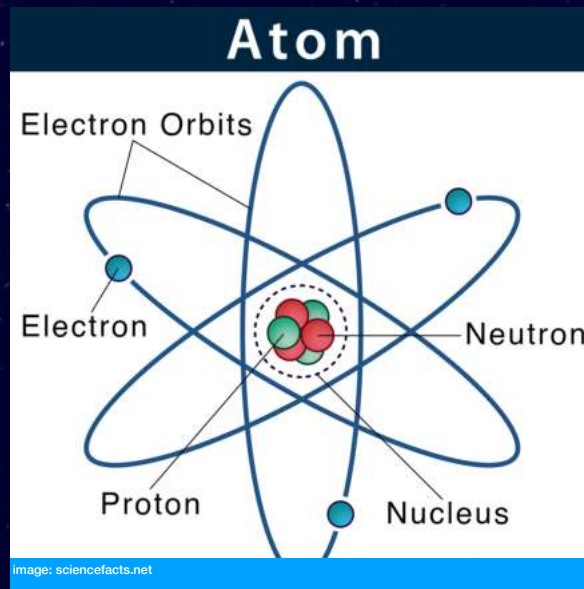
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part I

we can start with some history...

3

Starting point: the Atom

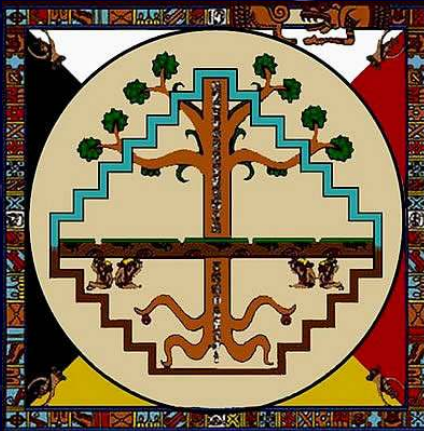


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ancient Mayans (10th BC)



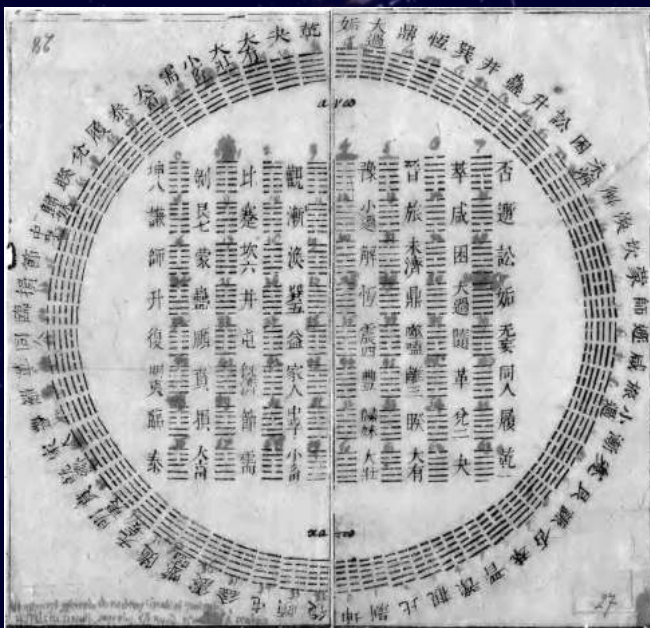
source: Duke U.



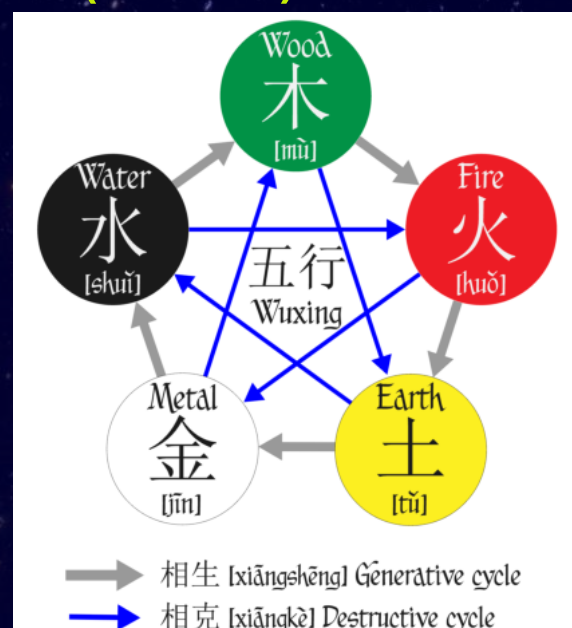
The Four Elements and Duality

- * **Elemental Forces:** Earth, Water, Fire, and Air, governed by the sacred center, often associated with the Fifth Element or the cosmos.
- * **Duality:** All matter was thought to exist in a state of dynamic balance hot and cold, light and dark, life and death. Matter and spirit were not separate

ancient Chinese (5th BC)



source: wikipedia

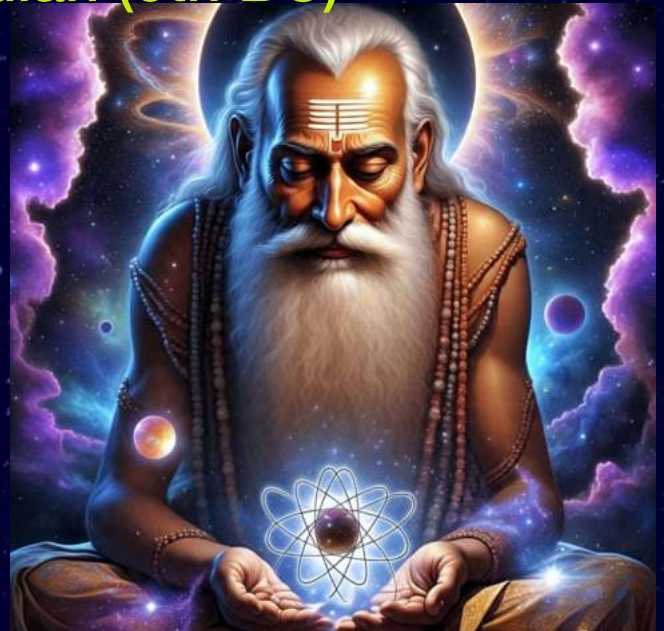


ancient Indian (6th BC)

The (Atomic) Theory of Anu and Parmanu

Acharya Kanad's Vaisheshika Sutra, a Sanskrit text, outlines his atomic theory. According to his theory:

- *All matter is made of indivisible, indestructible particles called **Parmanu** (subtle atoms).
- *These Parmanu combine to form **Anu** (visible atoms), which then create more complex matter.
- *The combination of these atoms occurs due to an **unseen force, driven by motion and time**.
- *Atoms differ based on the properties of the matter they create (such as taste, smell, texture).



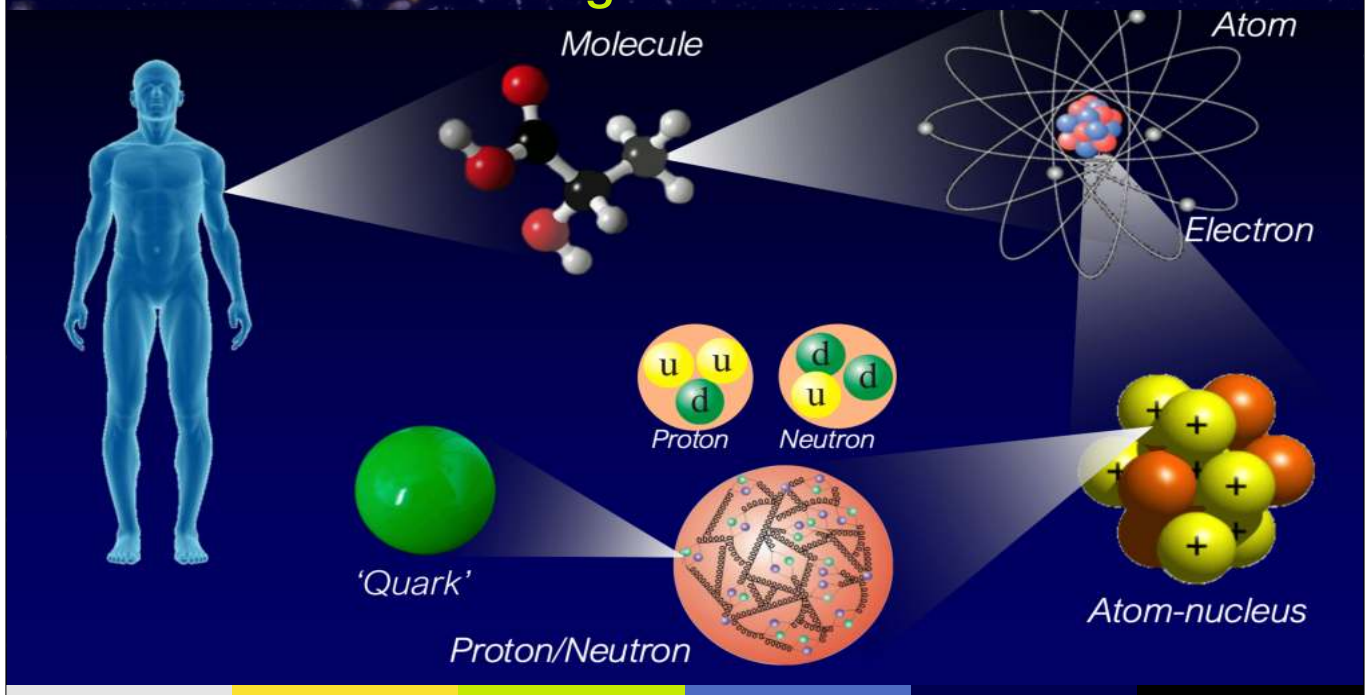
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ancient Greeks



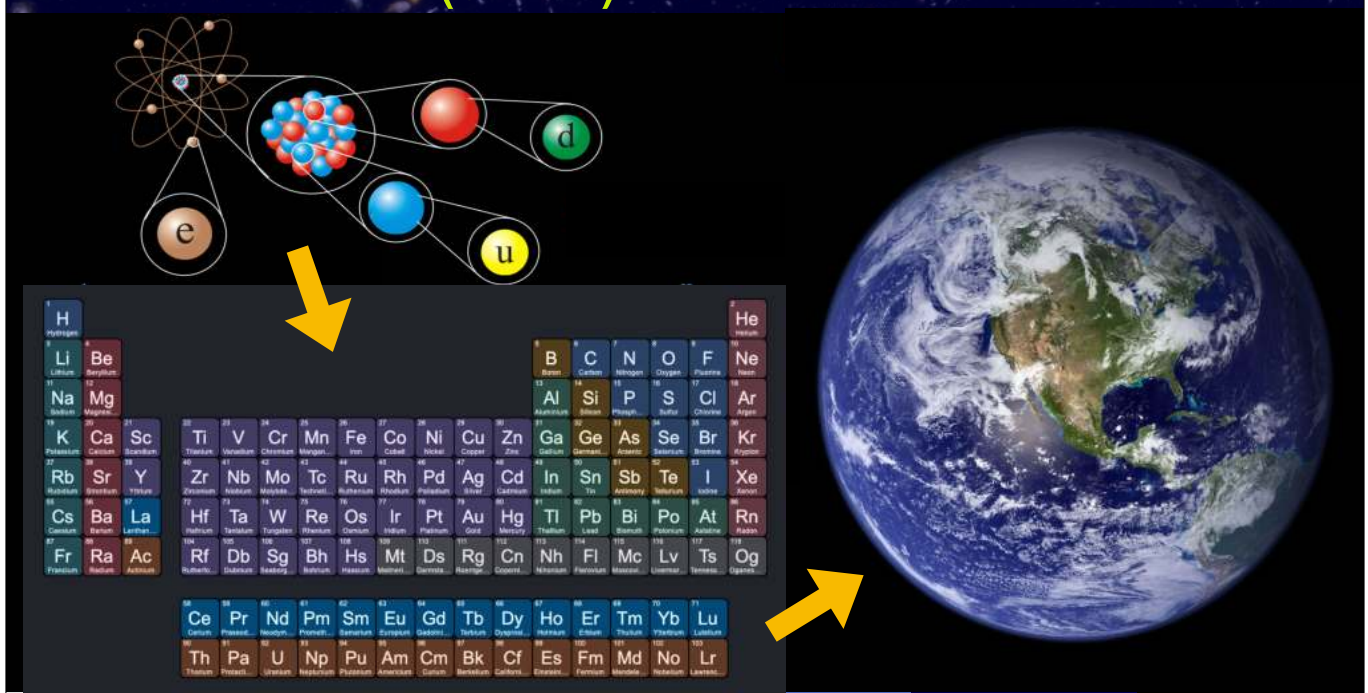
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Building blocks of matter

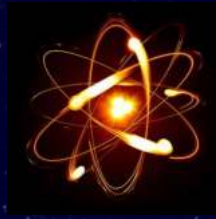
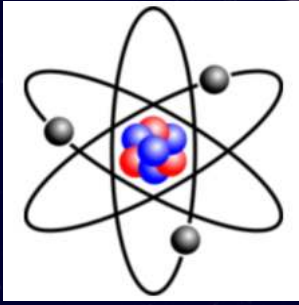


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(Stable) matter on Earth



10



- Does something “move”?
- Is it “empty”?
- How does an electron exist?

Also

- Quantum Mechanics!

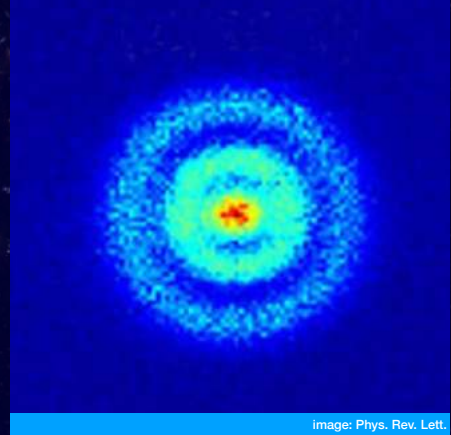


image: Phys. Rev. Lett.

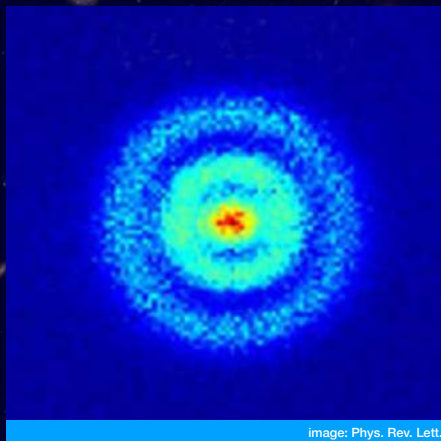
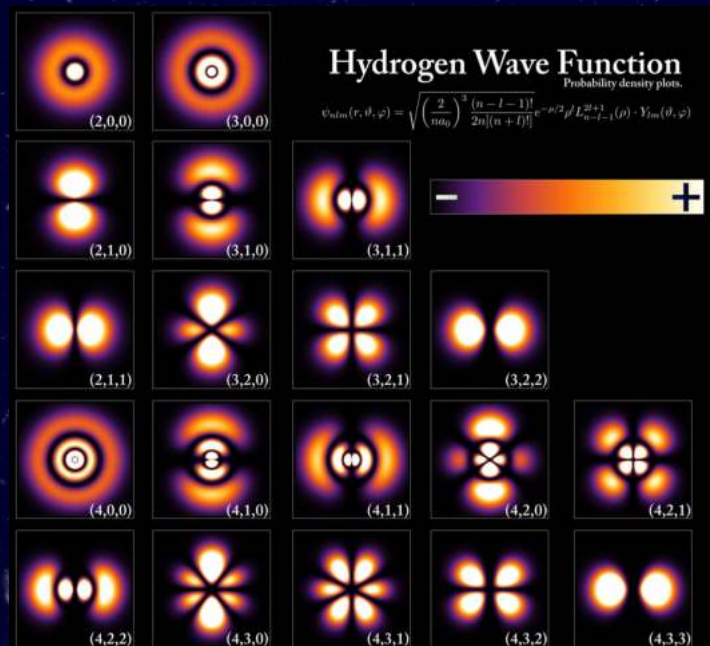


image: Phys. Rev. Lett.



- JJ Thomson (1897): Cathode rays are bent by electric and magnetic field

- Charged particles

1. Compensate Electric and Magnetic deflection:

$$F_E = F_B$$

$$qE = qvB ; v = E/B$$

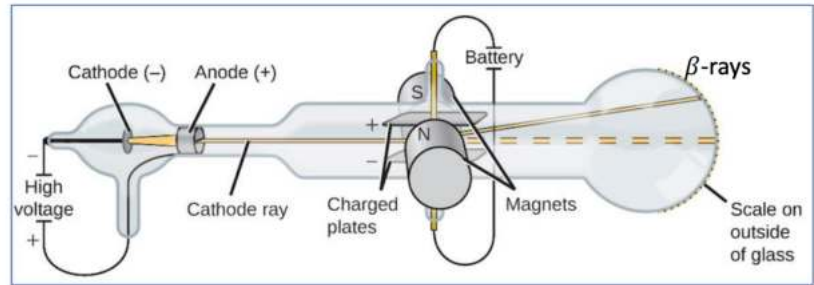
2. Only B-field, cycle orbit:

$$F_c = F_B$$

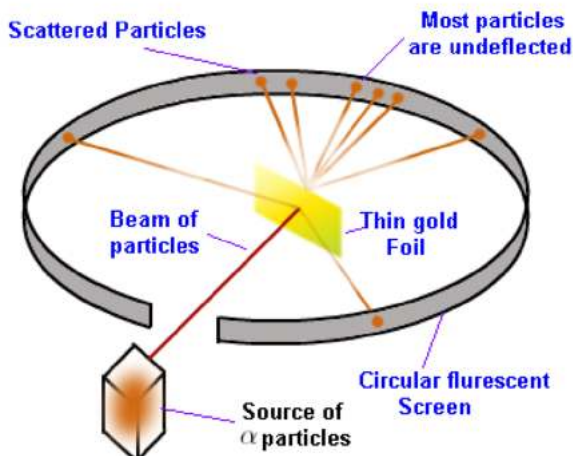
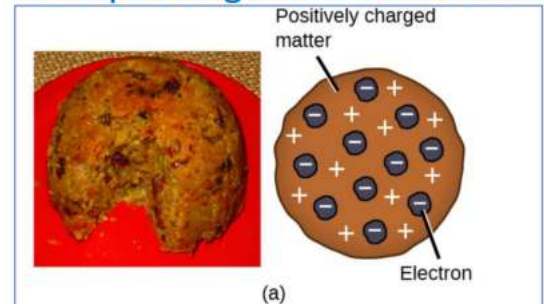
$$\frac{mv^2}{r} = qvB$$

$$\frac{q}{m} = \frac{E}{rB^2}$$

Electrons are much lighter than ions of same charge!



- Plum-pudding model of atoms:

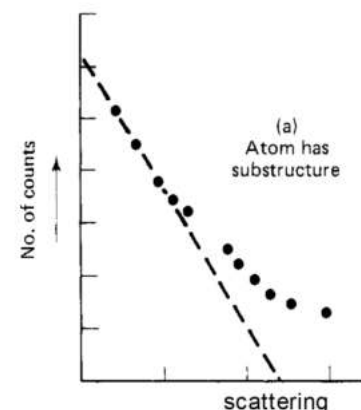
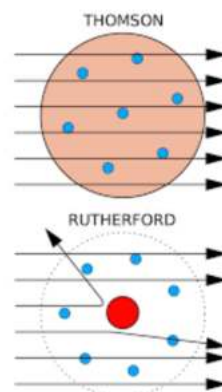


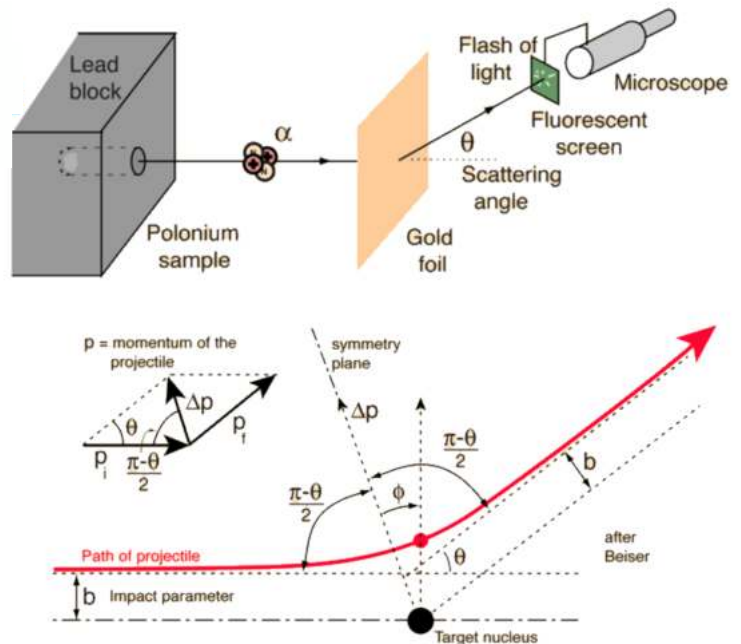
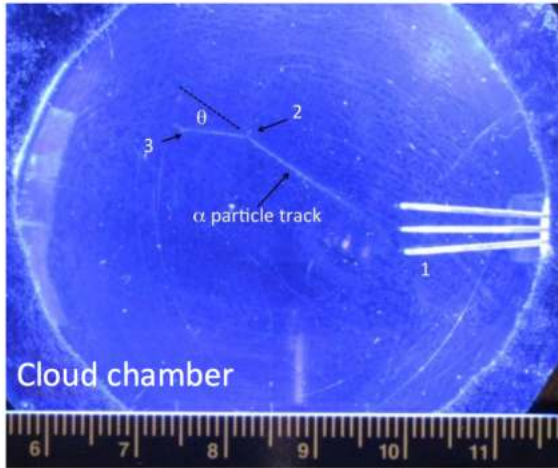
- Atom is basically just vacuum:

- Atom size: $\sim 10^{-10} \text{ m}$
- Nucleus size: $\sim 10^{-15} \text{ m}$

- Rutherford (1911)

- α -particles at gold target
- Most particle pass undisturbed, few have a "hard" collision
- Atom has substructure: small heavy nucleus



α 

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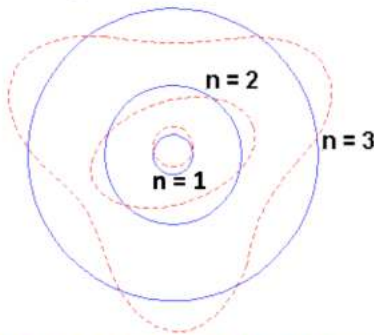
part II

atomic nuclei

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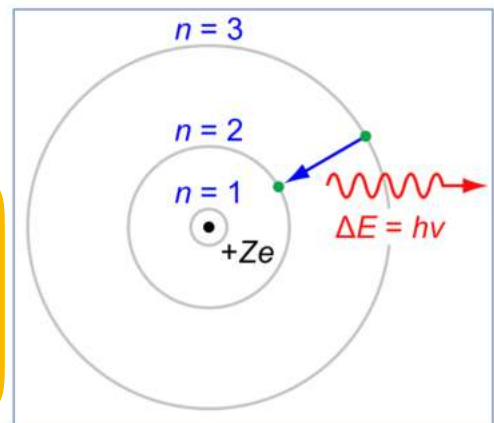
- Niels Bohr, 1914:

- Calculate atomic energy levels using semi classical method



$$L = rp = r \frac{h}{\lambda}$$

$$= rn \frac{h}{2\pi r} = n \frac{h}{2\pi} = n\hbar$$



Balmer series

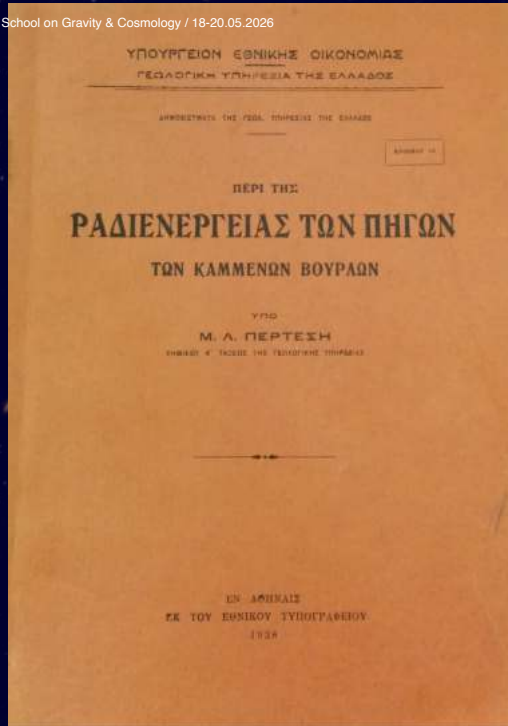
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- Nuclei include protons, p
 - Masses of nuclei studied in chemistry (Avogadro)
 - Hydrogen = 1 proton: $M_H = 1 \text{ GeV}/c^2 = 200 M_e$
- Study q/m of nuclei:
 - Masses of nuclei do not scale with charge, but with $A \approx 2 \times Z$
 - A = mass number, Z = atomic number = nucleus charge
 - What keeps heavy nuclei together?
- Chadwick 1932: Discovery of neutron, n
 - Neutron has almost same mass as proton
 - Neutrons act as a glue to nucleons in the nucleus
 - Later we will see it is due to the gluon color force
- Notation: ${}^A_Z\text{MyAtom}$, with $Z = \#p$, $A = \#(p + n)$
 - Isotopes: same $\#p$, different $\#n$



James Chadwick

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1926

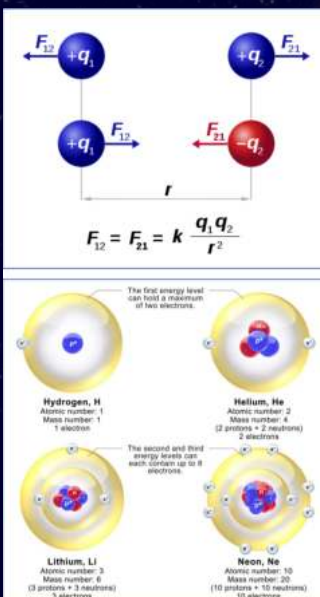
“...ο πυρήν του υδρογόνου φέρει τη μονάδα του θετικού ηλεκτρισμού, το όλον δε άτομον του υδρογόνου αποτελείται εκ του πυρήνος τούτου και ενός αρνητικού ηλεκτρονίου, περί τον πυρήνα αυτόν κινουμένου...”

“...ο πυρήν του ηλίου αποτελείται από τέσσαρας πυρήνας υδρογόνου και δύο αρνητικά ηλεκτρόνια...”

“...the nucleus of hydrogen carries one unit of positive charge, while the atom of hydrogen consists of the nucleus and one negative electron in orbit around it (the nucleus)...”

“...the nucleus of helium consists of four nuclei of hydrogen and two negative electrons...”

E/M vs nuclear force

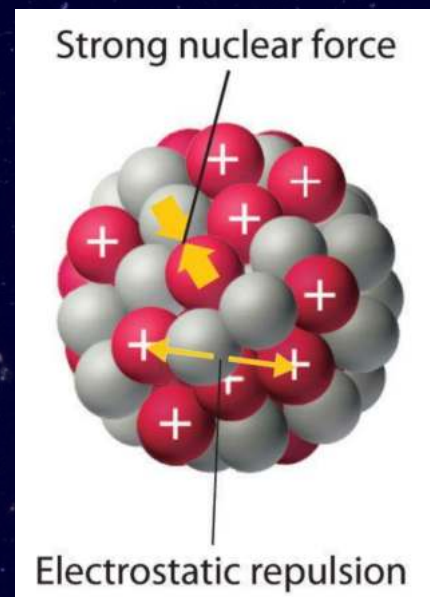


Electromagnetic force:

generated by exchanged massless photons

Strong nuclear force:

must be **stronger** than E/M **inside** the nucleus
must be **weaker** than E/M **outside** the nucleus



- Pauli exclusion principle:

Two fermions (p,n) cannot be at the same position (strong repulsion at $r \rightarrow 0$)

- Square well potential:

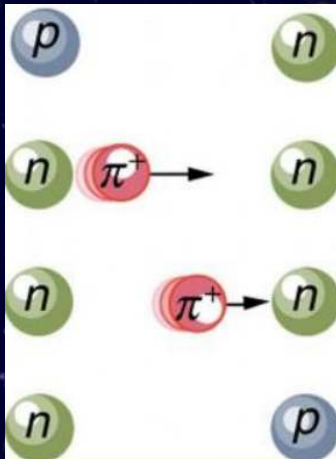
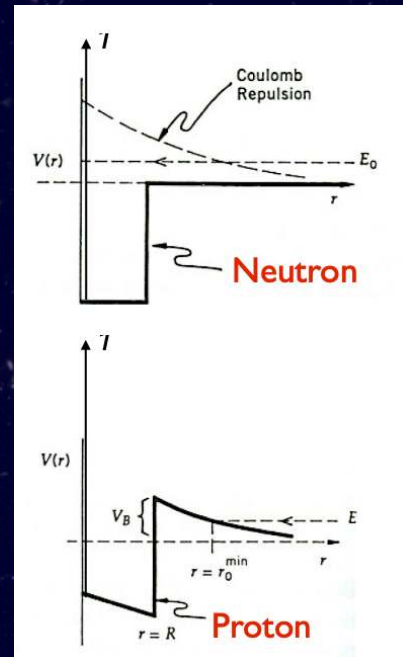
For p, a Coulomb barrier V_B exists

Can be neglected for $r \gg R$

R can be determined by scattering (RBS)

- But what is the mediator?

Yukawa postulates the “pion”



- Yukawa:

a π meson generates the attraction between nucleons

- π meson mass:

Estimated via the Heisenberg uncertainty principle

$$\Delta E \cdot \Delta t \leq \frac{\hbar}{2}$$

- What does it mean?

Energy conservation can be violated if the uncertainty principle is valid

$$E = mc^2 \quad \Delta t \sim \frac{\hbar}{mc^2} \rightarrow R \sim c\Delta t = \frac{\hbar}{2mc}$$

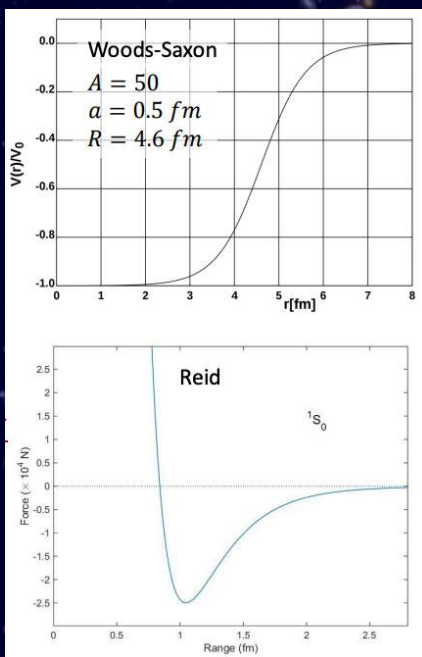
Photons: $m=0 \rightarrow$ infinite range
Pions: $m \neq 0 \rightarrow$ discrete range

A traditional goal of nuclear physics has been the understanding of the properties of atomic nuclei in terms of the “bare” interaction between pairs of nucleons. However, the underlying theory of strong interactions, QCD, shows that the NN interaction is not fundamental.

The nucleon-nucleon interaction is still unknown nowadays!

The NN interaction:

- * has short range ($\sim 1 \text{ fm} = 10^{-15} \text{ m}$) \leftarrow particle scattering, size of nucleus
- * is (very) attractive for distances 1-2.5 fm
- * is repulsive for short distances, $< 0.7 \text{ fm} \rightarrow$ solid, repulsive core; shapes the size of the nucleus
- * saturates; nucleons interact with their closest neighbors. Binding energy: 8 MeV/A
- * is charge independent; $V_{pn}=V_{nn}=V_{pp} \leftarrow$ deuteron
- * depends on relative spin orientation of nucleons
- * has a tensor character \rightarrow departs from spherical symmetry



• Woods Saxon Potential, 1954

- $V(r) = -\frac{V_0}{1 + \exp\left(\frac{r-R}{a}\right)}$
- Nuclear Radius: $R = 1.25 \text{ fm} \times A^{1/3}$
 A is mass number, $V_0 \approx 50 \text{ MeV}$, $a \approx 0.5 \text{ fm}$

• Reid Potential, 1968, (semi-empirical) implements repulsive core:

- $V_{\text{Reid}}(R) = -10.463 \frac{e^{-\mu r}}{\mu r} - 1650.6 \frac{e^{-4\mu r}}{\mu r} + 6482.2 \frac{e^{-7\mu r}}{\mu r}$
in MeV with $\mu = 0.7 \text{ fm}^{-1}$

Nijmegen potential:

- De Swart, Rijken et al., 1980 - 1990
- <https://arxiv.org/pdf/nucl-th/9509024.pdf>

Microscopic approach: The degrees of freedom are the nucleons (position, spin, isospin)

$$\vec{r}_i, \vec{p}_i, \vec{s}_i, \vec{t}_i \quad (i = 1, \dots, A) \quad \Psi_{NN} = \psi(\vec{r}) \chi_S^{m_s} \phi_T^{T_z}$$

$$H = \sum_{i=1}^A T_i + \frac{1}{2} \sum_{i,j} V(i,j)$$

Parameterizations of the NN potential rely on NN scattering and the deuteron properties.

There are two classes: local and non-local

$V(i,j)$ is not known; symmetry arguments help.

NNN terms are often neglected. In (very recent) years, this has been revised!

General form: $V(1,2) = V(\vec{r}_1, \vec{p}_1, \vec{\sigma}_1, \vec{\tau}_1, \vec{r}_2, \vec{p}_2, \vec{\sigma}_2, \vec{\tau}_2)$

Nucleon exchange invariance: $V(1,2) = V(2,1)$

Translational invariance: $V(1,2) = V(\vec{r}, \vec{p}_1, \vec{\sigma}_1, \vec{\tau}_1, \vec{p}_2, \vec{\sigma}_2, \vec{\tau}_2)$

Galilean invariance: $V(1,2) = V(\vec{r}, \vec{p}, \vec{\sigma}_1, \vec{\tau}_1, \vec{\sigma}_2, \vec{\tau}_2)$

Rotation invariance: $V(1,2) = \text{scalar}$

Parity invariance: $V(\vec{r}, \vec{p}, \vec{\sigma}_1, \vec{\tau}_1, \vec{\sigma}_2, \vec{\tau}_2) = V(-\vec{r}, -\vec{p}, \vec{\sigma}_1, \vec{\tau}_1, \vec{\sigma}_2, \vec{\tau}_2)$

Time-reversal invariance: $V(\vec{r}, \vec{p}, \vec{\sigma}_1, \vec{\tau}_1, \vec{\sigma}_2, \vec{\tau}_2) = V(\vec{r}, -\vec{p}, -\vec{\sigma}_1, \vec{\tau}_1, -\vec{\sigma}_2, \vec{\tau}_2)$

Under the symmetry considerations:

$$V(1,2) = V_C + V_S(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T S_{12}(\vec{r}) + V_{T'} S_{12}(\vec{p}) + V_L(\vec{L} \cdot \vec{S}) + V_Q(\vec{L} \cdot \vec{S})^2$$

spin-orbit interaction:
$$\vec{L} \cdot \vec{S} = \frac{1}{2} (\vec{r} \times \vec{p}) \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)$$

tensor operator:
$$S_{12} = 3 \left(\vec{\sigma}_1 \cdot \frac{\vec{r}}{r} \right) \left(\vec{\sigma}_2 \cdot \frac{\vec{r}}{r} \right) - (\vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

scalar terms:
$$V_\alpha = V_\alpha(r^2, p^2, L^2) \quad \alpha = \{C, S, T, T', LS, Q\}$$

Klein-Gordon for the pion field:
$$\left(\nabla^2 - \frac{m_\pi^2 c^2}{\hbar^2} \right) \phi = -g \delta(\vec{r})$$

solution:
$$\phi = g \frac{e^{-\mu r}}{r} \quad \mu = \frac{m_\pi c}{\hbar}$$

Yukawa potential:
$$V = g^2 \frac{e^{-\mu r}}{r}$$

The Yukawa potential function,

$$V_{\text{Yukawa}}(r) = -g^2 \frac{e^{-kmr}}{r},$$

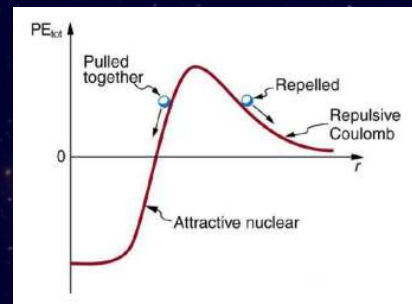
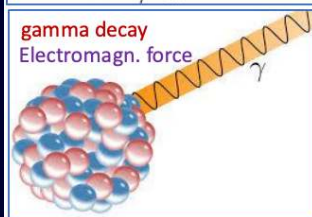
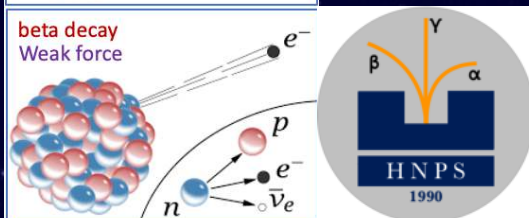
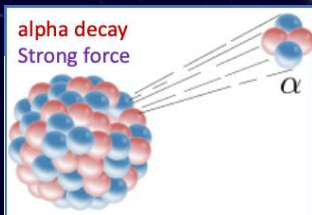
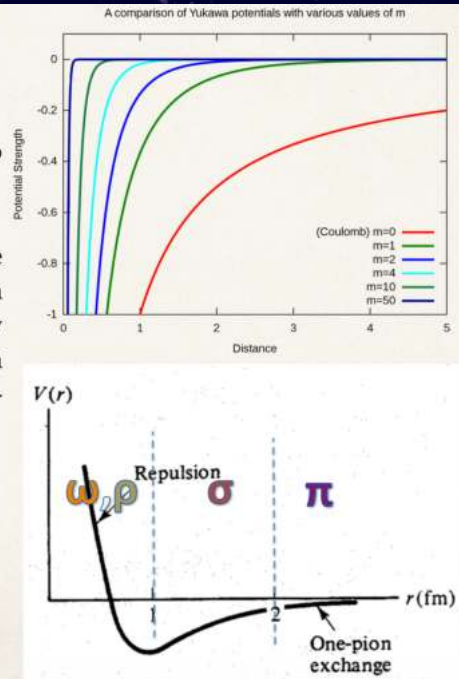
$V_{\text{Yukawa}}(r) \rightarrow -\infty$ as $r \rightarrow 0$ (same as the Coulomb potential).

This “unnatural” behavior of the infinitely attractive force is corrected by the addition of a potential of an impenetrable hard sphere (i.e. an infinitely repulsive force), the existence of which has been ascertained by scattering experiments p - p and p - n.

Thus, the N-N potential takes the form:

$$V(r) = +\infty, \quad \forall r < r_c$$

$$V(r) = V_{\text{Yukawa}}(r), \quad \forall r > r_c$$



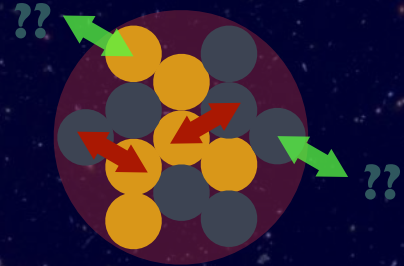
- Unstable nucleus
- Quantum tunneling process

$$N = N(0)e^{-\lambda t} = N(0)e^{-t/\tau} = N(0)e^{-t \ln 2/t_{1/2}}$$

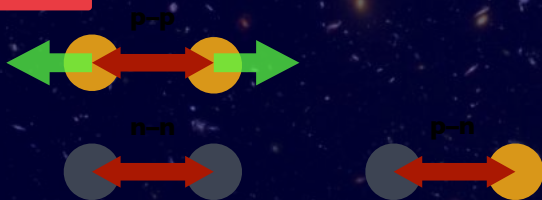
$BE \sim A$ εξαρτάται από τον όγκο



$BE \sim -A^{2/3}$ εξαρτάται αρνητικά από την επιφάνεια



$BE \sim -Z^2 / A^{1/3}$ εξαρτάται αρνητικά από τη δύναμη Coulomb



$BE \sim -(N-Z)^2 / A = -(A-2Z)^2 / A$

εξαρτάται από την ασυμμετρία Z-N

$BE \sim \delta$, $\delta = +1, 0, -1$ για e-e, e-o, o-o

εξαρτάται από το ζεύγος

$BE(Z,A) = a_v A$ Όγκος

$- a_s A^{2/3}$ Επιφάνεια

$- a_c Z^2 / A^{1/3}$ Coulomb

$- a_{sym} (A-2Z)^2 / A$ Συμμετρία

$- \delta a_p / A^{1/2}$ Ζεύγος

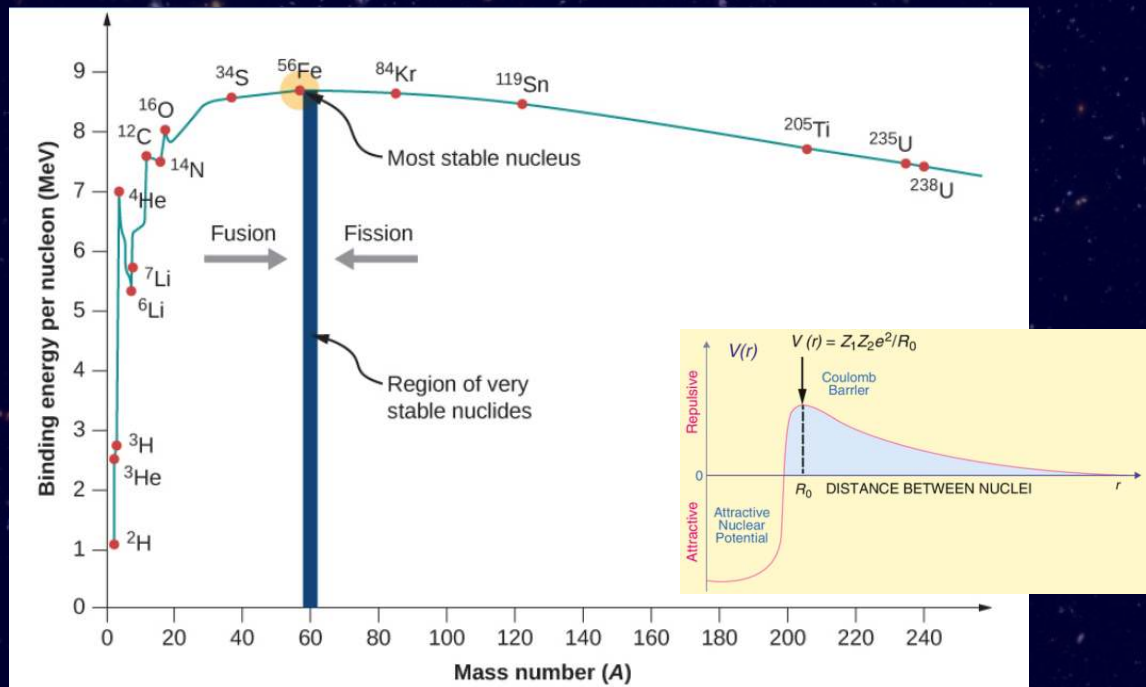


Carl Friedrich von Weizsäcker

Atomic Mass Evaluation 2022

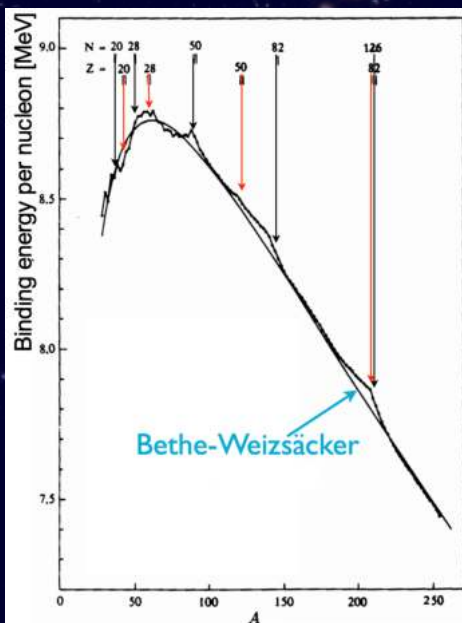
a_v	15.247 MeV
a_s	16.325 MeV
a_c	0.685 MeV
a_{sym}	22.098 MeV
a_p	10.608 MeV

Binding Energy, $B(A,Z)$



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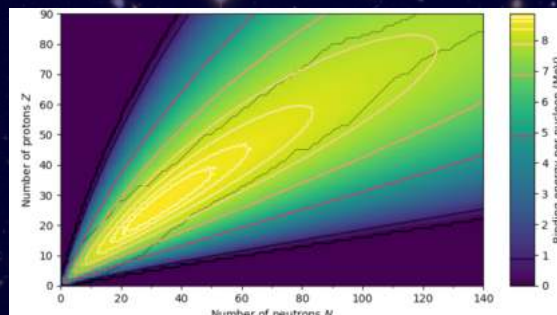
"magic" numbers



- $B(A,Z)/A$ fits the data quite well
- It cannot explain 'peaks' that occur at 'magic numbers', where those nuclei are strongly bound
- N or $Z = 2, 8, 20, 28, 50, 82, 126$

Double magic nuclei (both N and Z are magic):

- $^4\text{He}, ^{16}\text{O}, ^{40}\text{Ca}, ^{48}\text{Ca}, ^{48}\text{Ni}, ^{208}\text{Pb}$



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There are many «*structure effects*» in nuclei, that **can not be reproduced by macroscopic approaches** like the liquid drop model.

Need for **microscopic approaches**, for which the fundamental ingredients are the nucleons and the interaction between them

Nucleus = A nucleons in strong interaction

The many-body problem
Can be solved exactly for $A < 4$
For $A \gg 10$: approximations

Nucleon-Nucleon force
unknown

Different effective forces used depending on the method chosen to solve the many-body problem

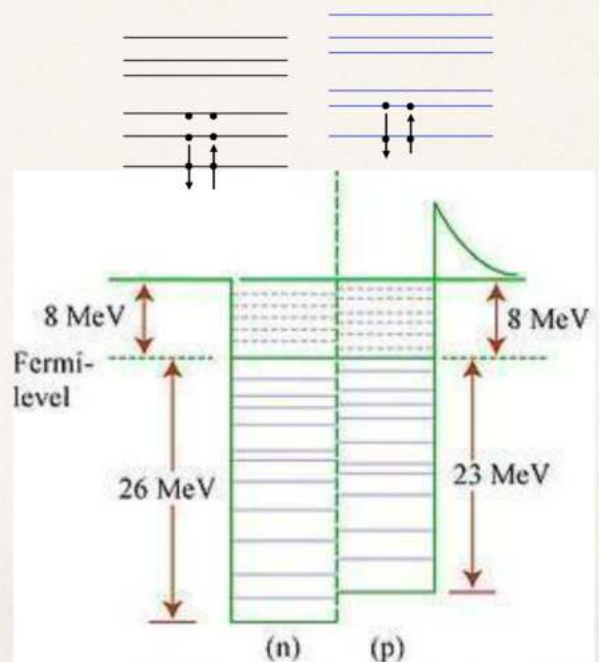
Shell Model

only a small number of particles are active

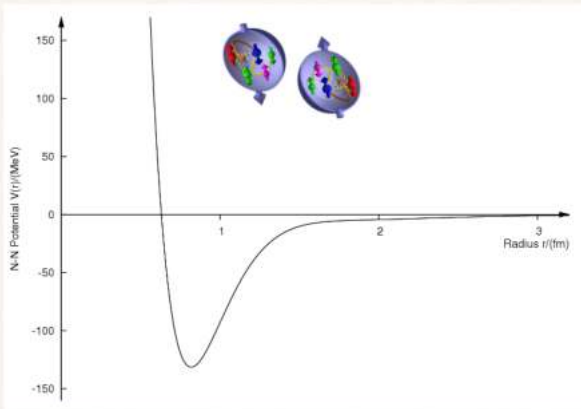
Mean Field models

- no inert core
- not all the correlations between particles are taken into account

- ✦ **Nucleons are quantum objects**: only some values of the energy are available, a **discrete number of states**
- ✦ **Nucleons are fermions**: two nucleons can not occupy the same quantum state: the Pauli principle. **The energy levels of the nucleons are filled from the lowest to the highest** as nucleons are added to the nucleus.
- ✦ There are **separate energy levels for protons and neutrons**.
- ✦ Nucleons **fill every energy level in orbitals with a definite angular momentum**.
- ✦ As in atoms, **many nuclear properties** (angular momentum, magnetic moment, shape, etc.) **are determined by the last occupied or unoccupied level**.



The basic assumption of the nuclear shell model is, to first order, the independent motion of each nucleon (proton or neutron) in an average nuclear potential.



... model Hamiltonian for A independent nucleons:

$$H_0 = \sum_{i=1}^A (T_i + U_i(r)) \equiv \sum_{i=1}^A h_0(i)$$

eigenfunctions: $\Psi_{a_1, a_2, \dots, a_A}(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A) = \prod_{i=1}^A \phi_{a_i}(\vec{r}_i)$

eigenvalues: $E_0 = \sum_{i=1}^A \epsilon_{a_i}$

... antisymmetrization:

$$\Psi_{a_1, a_2}(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}} [\phi_{a_1}(\vec{r}_1) \phi_{a_2}(\vec{r}_2) - \phi_{a_1}(\vec{r}_2) \phi_{a_2}(\vec{r}_1)]$$

The average potential $U(r)$ is not given explicitly. If one starts from a one- plus two-body Hamiltonian:

$$H = \sum_{i=1}^A T_i + \frac{1}{2} \sum_{i,j=1}^A V_{i,j}$$

The smaller the effect of H_{RES} , the better the assumption of an average, independent field for each nucleon.

$$H = \sum_{i=1}^A [T_i + U_i(r_i)] + \left(\frac{1}{2} \sum_{i,j=1}^A V_{i,j} - \sum_{i=1}^A U_i(r_i) \right) \equiv \sum_{i=1}^A h_0(i) + H_{RES}$$

Nuclear potential deduced from data:

Wood Saxon potential: $V(r) = -\frac{V_0}{1 + \exp[(r - R)/a]}$

Square well: $V(r) = -V_0 (r \leq R)$ and $V(r) = 0 (r > R)$

harmonic oscillator: $V(r) = \frac{1}{2} m \omega^2 r^2$

... solutions for the HO potential:

$$U(r) = \frac{1}{2} m \omega^2 r^2$$

$$u_{kl}(r) = N_{kl} r^{l+1} e^{-\nu r^2} L_k^{l+1/2}(2\nu r^2) \quad (\nu = m\omega/2\hbar)$$

Laguerre polynomials

$$E = \hbar\omega(2k + l + \frac{3}{2}) = \hbar\omega(N + \frac{3}{2})$$

$N = 0, 1, 2, \dots$ major oscillator quantum number
 $l = N, N-2, \dots, 1$ or 0 orbital quantum number
 $k = (N-l)/2$ radial quantum number

$n = k+1 = (N-l+2)/2$ number of nodes of the radial wave function in the interval $[0, \infty)$.

$$D_N = 2 \sum_{l=0 \text{ or } 1}^N (2l+1) = (N+1)(N+2)$$

spin projections

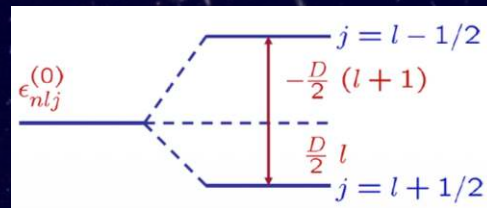
The spin-orbit coupling

Mayer (1949,1950) and Haxel, Jensen, Suess (1949, 1950) – the average single-nucleon potential should contain a spin-orbit term

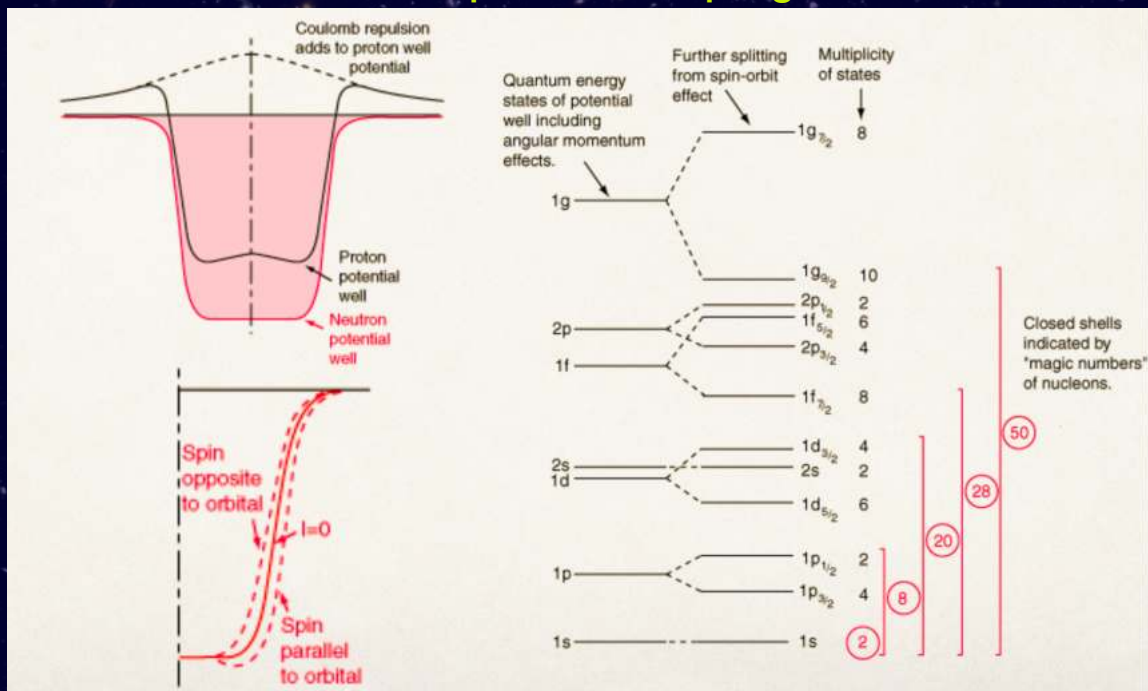
$$h = h_0 + \zeta(r) \vec{l} \cdot \vec{s}$$

... an intrinsically relativistic effect. It is automatically included in the effective potential when the single – nucleon dynamics is described by the Dirac equation. When the nucleons are described as non-relativistic particles, the spin-orbit term must be added to the Schrödinger equation.

$$\begin{aligned} \Delta \varepsilon_{nlj=l+1/2} &= \frac{D}{2} l \\ \Delta \varepsilon_{nlj=l-1/2} &= -\frac{D}{2} (l+1) \end{aligned}$$

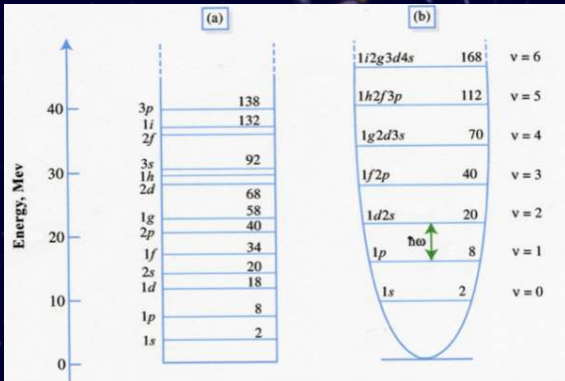


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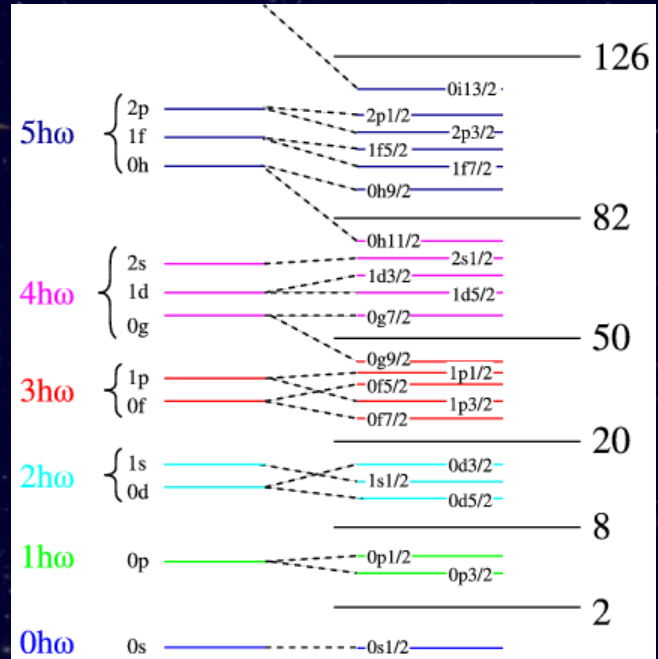
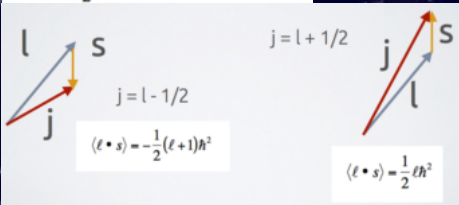
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Nuclear Shell Model



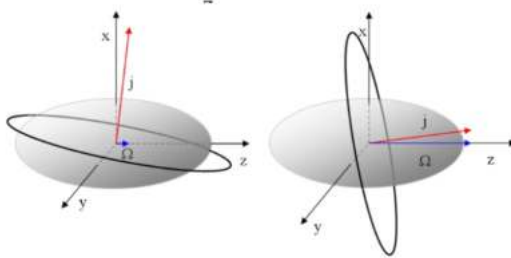
$$j^2 = (l + s)^2 = l^2 + s^2 + 2l \cdot s$$

$$\langle l \cdot s \rangle = \frac{1}{2} [j(j+1) - l(l+1) - s(s+1)] \hbar^2$$

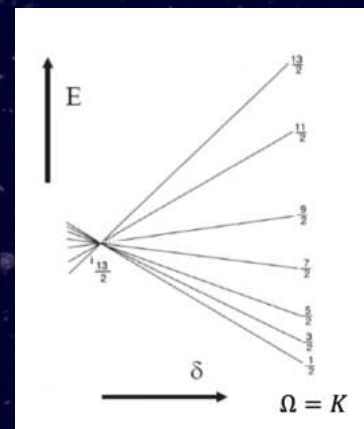


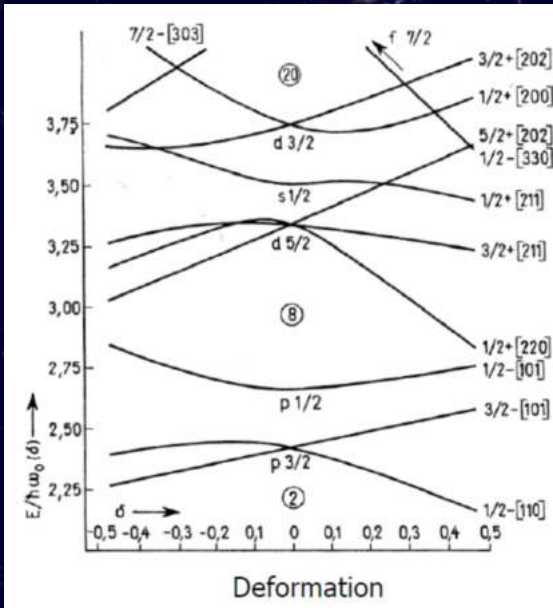
Deformed Shell Model

- spherical nucleus: $R = R_0$
- deformed nucleus: $R = R_0 \cdot \left[1 + \sum_{\lambda=2, \mu} \alpha_{2, \mu} \cdot Y_{2, \mu} \right]$
→ can rotate



- Separation of laboratory system and body-fixed (intrinsic) system
- $\Omega = K$ projection of the single-particle angular momentum onto the symmetry axis
- Rotation perpendicular to the symmetry axis will not change the Ω -quantum number





Nilsson Model is a single-particle model for deformed nuclei.

$$H = \frac{p^2}{2m} + \frac{m \cdot [\omega_x^2(x^2 + y^2) + \omega_z^2 \cdot z^2]}{2} + C \cdot \vec{L} \cdot \vec{S} + D \cdot \vec{L}^2$$

with $\omega_x^2 = \omega_0^2 \cdot \left(1 + \frac{2}{3} \cdot \delta\right)$ $\omega_z^2 = \omega_0^2 \cdot \left(1 - \frac{4}{3} \cdot \delta\right)$

The labelling of the Eigen-states is: $\Omega^\pi[Nn_z\Lambda]$

Ω projection of the total particle angular momentum on the symmetry axis

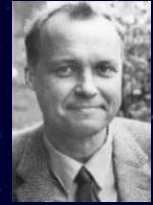
π parity of the wave function $\pi = (-1)^N$

N the principal quantum number of the major oscillator shells
 n_z the number of quanta associated with the wave function moving along the z-direction

$\Lambda = m_\ell$ projection of the orbital angular momentum onto the z-axis

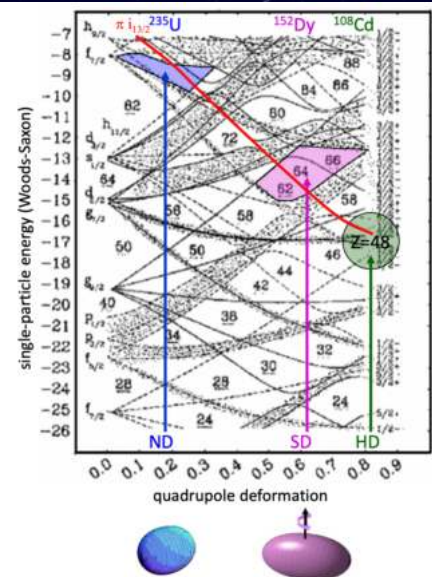
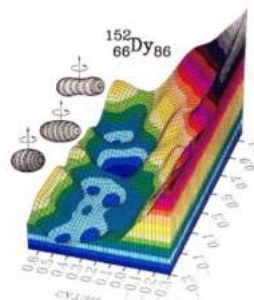
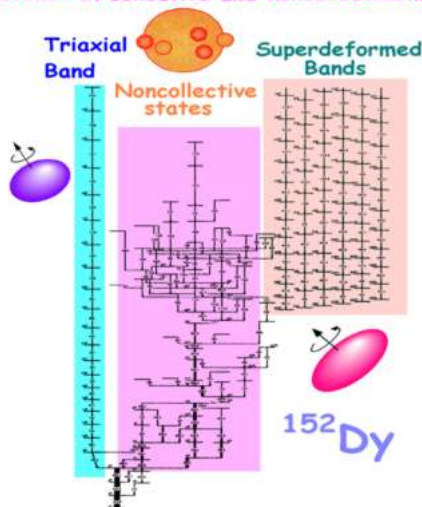
$$H = \underbrace{-\frac{\hbar^2}{2m} \Delta + \frac{m}{2} \omega_0^2 r^2 + C \cdot \vec{L} \cdot \vec{S} + D \cdot \vec{L}^2}_{\text{shell model with H.O. potential}} - m\omega_0^2 r^2 \delta \frac{4}{3} \sqrt{\frac{4\pi}{5}} Y_{20}(\theta, \Phi)$$

H_{def}



Sven Gösta Nilsson

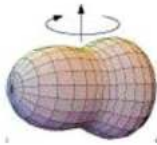
Coexistence of collective and noncollective motion



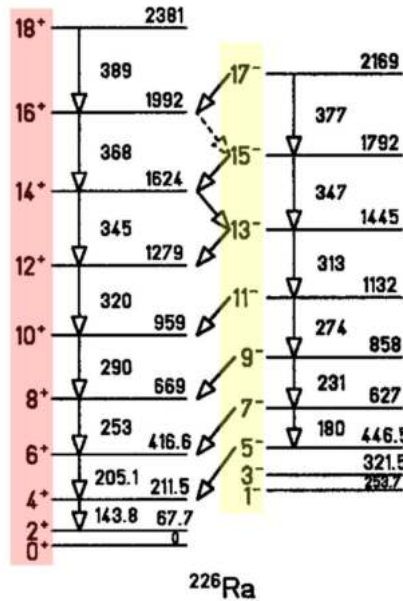
$$|\Psi\rangle = |\text{prolate}\rangle$$

$$P|\Psi\rangle = |\text{prolate}\rangle$$

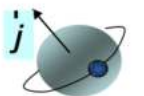
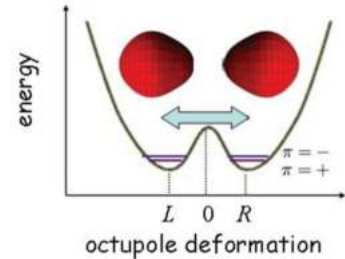
$$P|\Psi\rangle \neq |\Psi\rangle$$



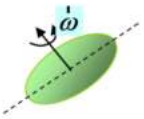
Rotation



Search of electric dipole moments (violation of the time reversal)

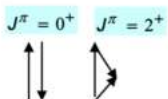


Spherical Symmetry: 1949 Mayer

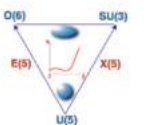


Deformed nuclear field (spontaneous symmetry breaking)
symmetry restoration → rotational spectra:
1952 Bohr-Mottelson

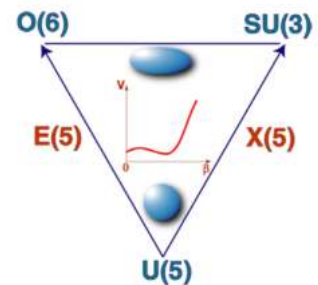
SU(3) dynamical Symmetry: 1958 Elliott

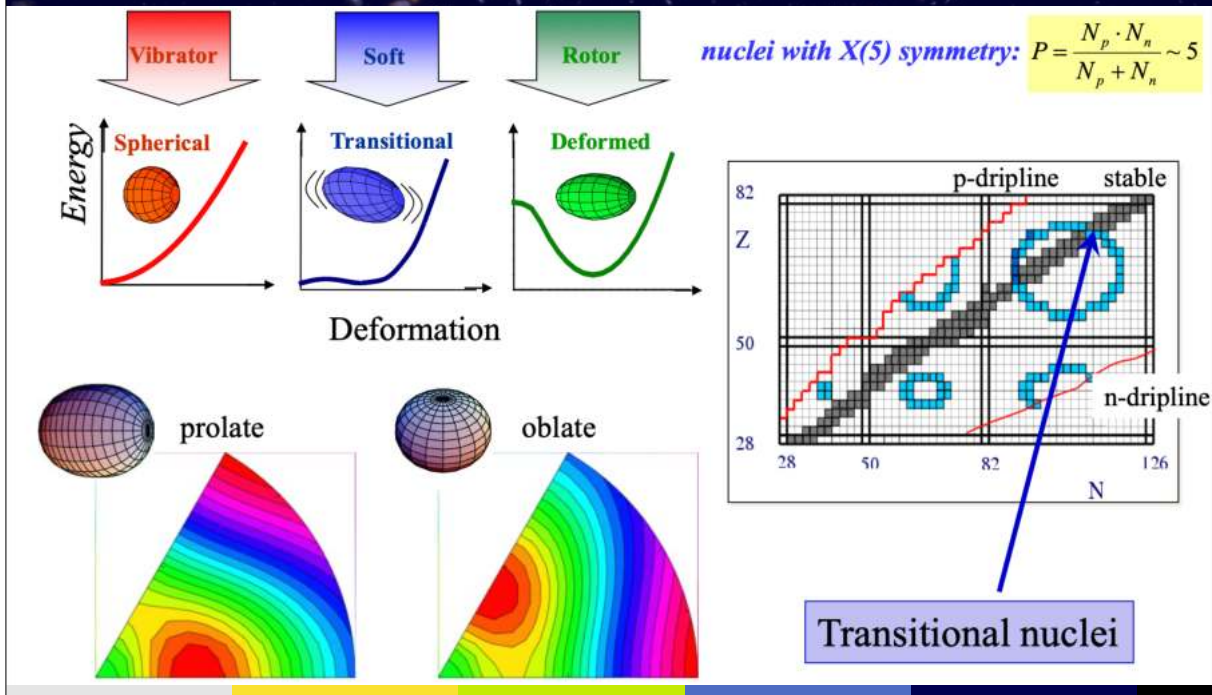


Interacting Boson Model (IBM dynamical symmetry):
1974 Arima and Iachello



Critical point symmetry E(5), X(5)
2000... F. Iachello





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part III

particles

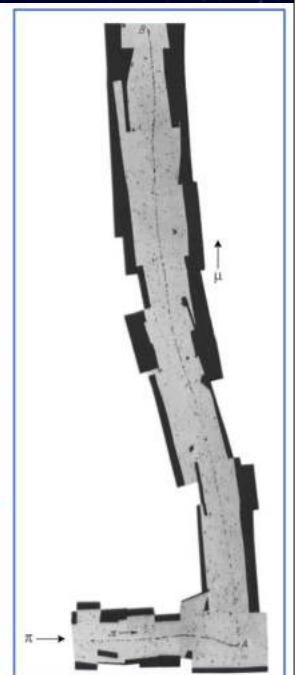
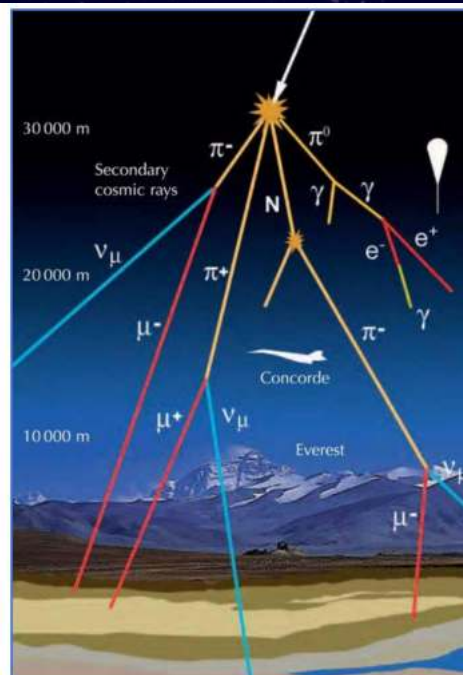
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Standard Model of Elementary Particles						interactions / force carriers (elementary bosons)	
three generations of matter (elementary fermions)			three generations of antimatter (elementary antifermions)				
I	II	III	I	II	III		
mass charge spin							
$\approx 2.2 \text{ MeV}/c^2$ $\frac{2}{3}$ $\frac{1}{2}$	$\approx 1.28 \text{ GeV}/c^2$ $\frac{2}{3}$ $\frac{1}{2}$	$\approx 173.1 \text{ GeV}/c^2$ $\frac{2}{3}$ $\frac{1}{2}$	$\approx 2.2 \text{ MeV}/c^2$ $-\frac{2}{3}$ $\frac{1}{2}$	$\approx 1.28 \text{ GeV}/c^2$ $-\frac{2}{3}$ $\frac{1}{2}$	$\approx 173.1 \text{ GeV}/c^2$ $-\frac{2}{3}$ $\frac{1}{2}$	0 0 1	$\approx 124.97 \text{ GeV}/c^2$ 0 0
u up	c charm	t top	\bar{u} antiup	\bar{c} anticharm	\bar{t} antitop	g gluon	H higgs
$\approx 4.7 \text{ MeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$	$\approx 96 \text{ MeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$	$\approx 4.18 \text{ GeV}/c^2$ $-\frac{1}{3}$ $\frac{1}{2}$	$\approx 4.7 \text{ MeV}/c^2$ $\frac{1}{3}$ $\frac{1}{2}$	$\approx 96 \text{ MeV}/c^2$ $\frac{1}{3}$ $\frac{1}{2}$	$\approx 4.18 \text{ GeV}/c^2$ $\frac{1}{3}$ $\frac{1}{2}$	0 0 1	
d down	s strange	b bottom	\bar{d} antidown	\bar{s} antistrange	\bar{b} antibottom	γ photon	
$\approx 0.511 \text{ MeV}/c^2$ -1 $\frac{1}{2}$	$\approx 105.66 \text{ MeV}/c^2$ -1 $\frac{1}{2}$	$\approx 1.7768 \text{ GeV}/c^2$ -1 $\frac{1}{2}$	$\approx 0.511 \text{ MeV}/c^2$ 1 $\frac{1}{2}$	$\approx 105.66 \text{ MeV}/c^2$ 1 $\frac{1}{2}$	$\approx 1.7768 \text{ GeV}/c^2$ 1 $\frac{1}{2}$	0 0 1	
e electron	μ muon	τ tau	e^+ positron	$\bar{\mu}$ antimuon	$\bar{\tau}$ antitau	Z Z ⁰ boson	
$< 2.2 \text{ eV}/c^2$ 0 $\frac{1}{2}$	$< 0.17 \text{ MeV}/c^2$ 0 $\frac{1}{2}$	$< 18.2 \text{ MeV}/c^2$ 0 $\frac{1}{2}$	$< 2.2 \text{ eV}/c^2$ 0 $\frac{1}{2}$	$< 0.17 \text{ MeV}/c^2$ 0 $\frac{1}{2}$	$< 18.2 \text{ MeV}/c^2$ 0 $\frac{1}{2}$	1 1 1	
ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	$\bar{\nu}_e$ electron antineutrino	$\bar{\nu}_\mu$ muon antineutrino	$\bar{\nu}_\tau$ tau antineutrino	W⁺ W ⁺ boson	W⁻ W ⁻ boson

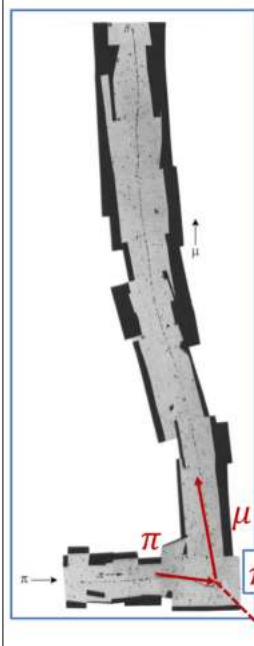
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Discovery of the pion

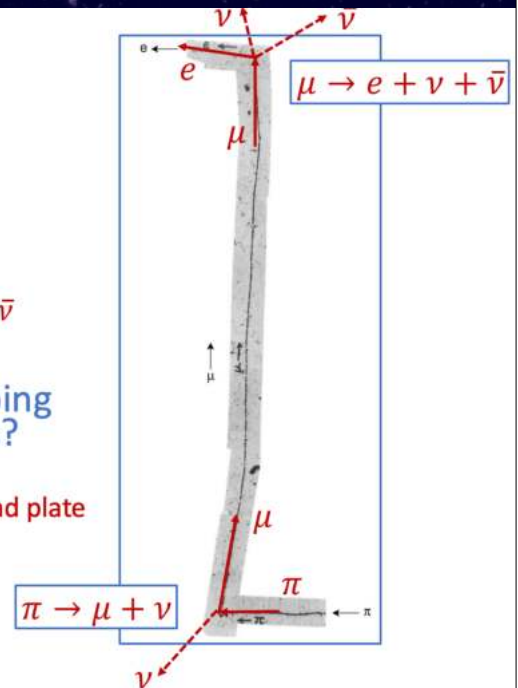
- Powell, 1947
 - Go to mountain top
 - Photographic emulsion
- Observes that cosmic rays include muons and pions
 - $m_\pi = 140 \text{ MeV}/c^2$
 - $m_\mu = 105 \text{ MeV}/c^2$
- A pion can decay into a muon, which again decays into an electron
- The pion was Yukawa's meson



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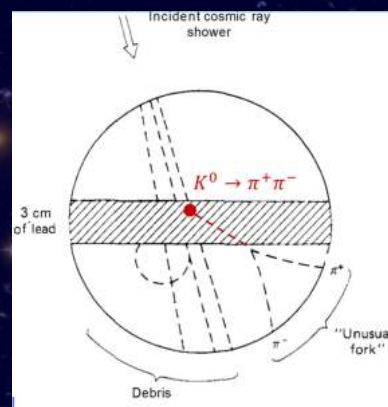
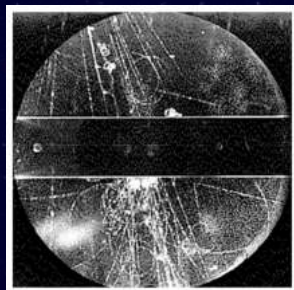


- Powell's discovery of muon and pion
 - Pion decay: $\pi \rightarrow \mu + \nu$
- Later discovery of Powell
 - Pion and muon decays: $\pi \rightarrow \mu + \nu$ followed by: $\mu \rightarrow e + \nu + \bar{\nu}$
- Neutrinos: are they a bookkeeping device or are they real particles?
 - Very weak interaction
 - Can penetrate 1000 lightyears thick lead plate
 - What is their mass? $m_\nu \neq 0$?



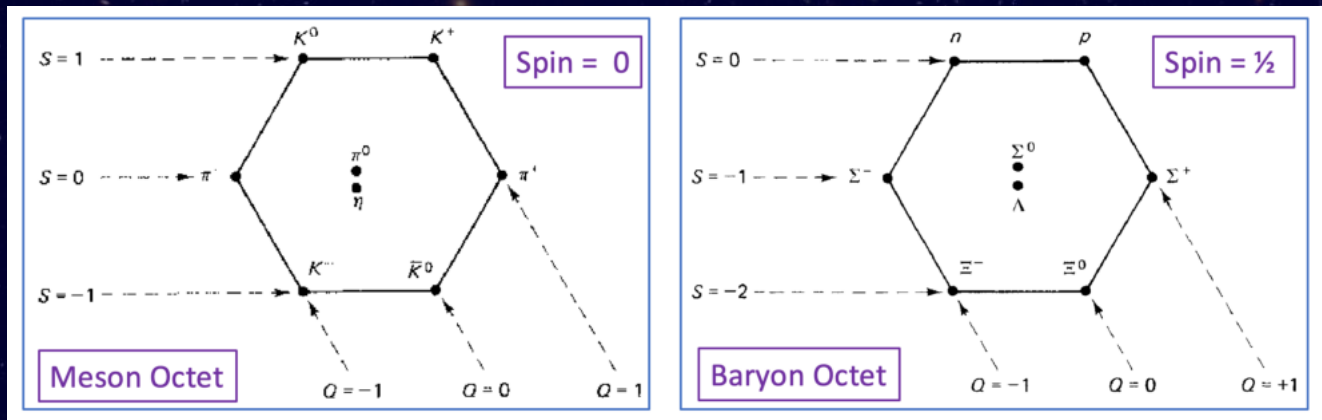
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- Strange: produced copiously, but decay slowly!
 - Abraham Pais: They are produced with the strong force, but they decay with the weak force.

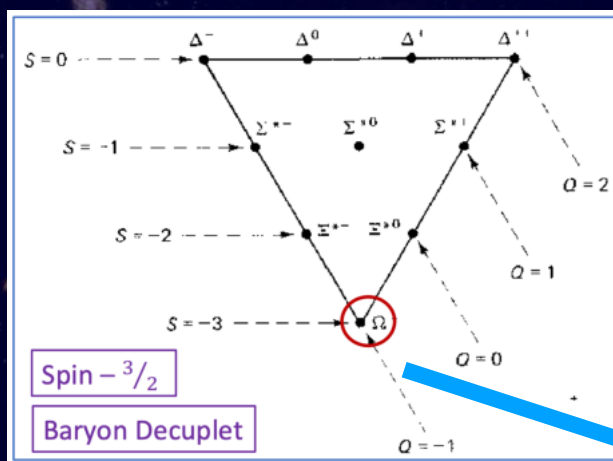


- Also observed: $K^+ \rightarrow \pi^+ + \pi^+ + \pi^- \rightarrow M_K \sim 500 \text{ MeV}$; it's a meson
- 1952, Brookhaven cosmotron: new strange baryons: Σ ($\sim 1.2 \text{ GeV}$), Ξ ($\sim 1.3 \text{ GeV}$)

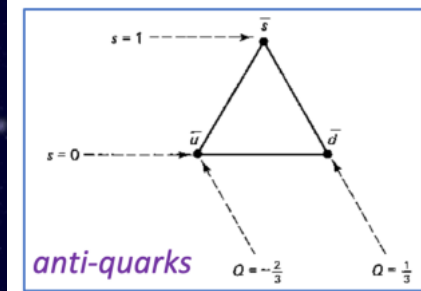
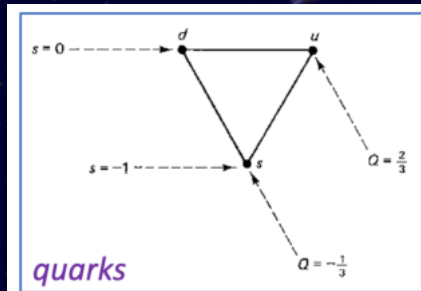
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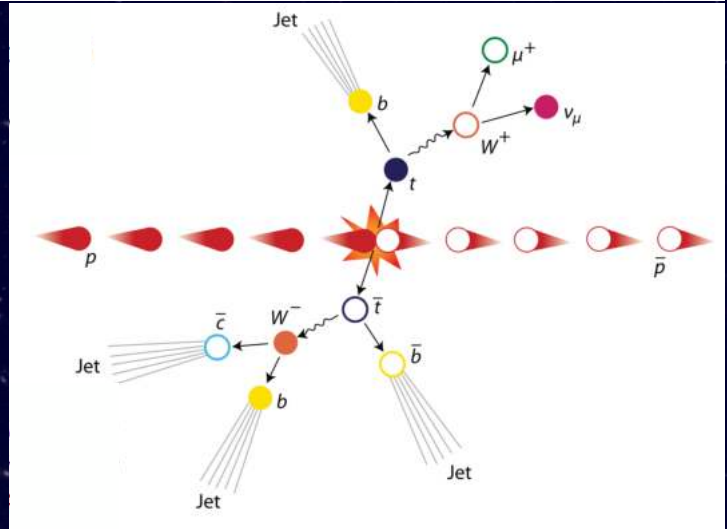


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Terminology: upness, downness, strangeness, charm, beauty = "Flavour"

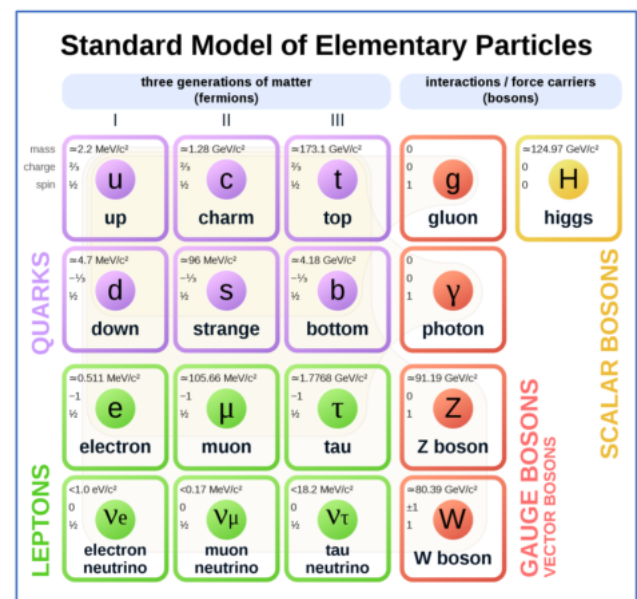
- Quarks have charge, flavor and color quantum numbers



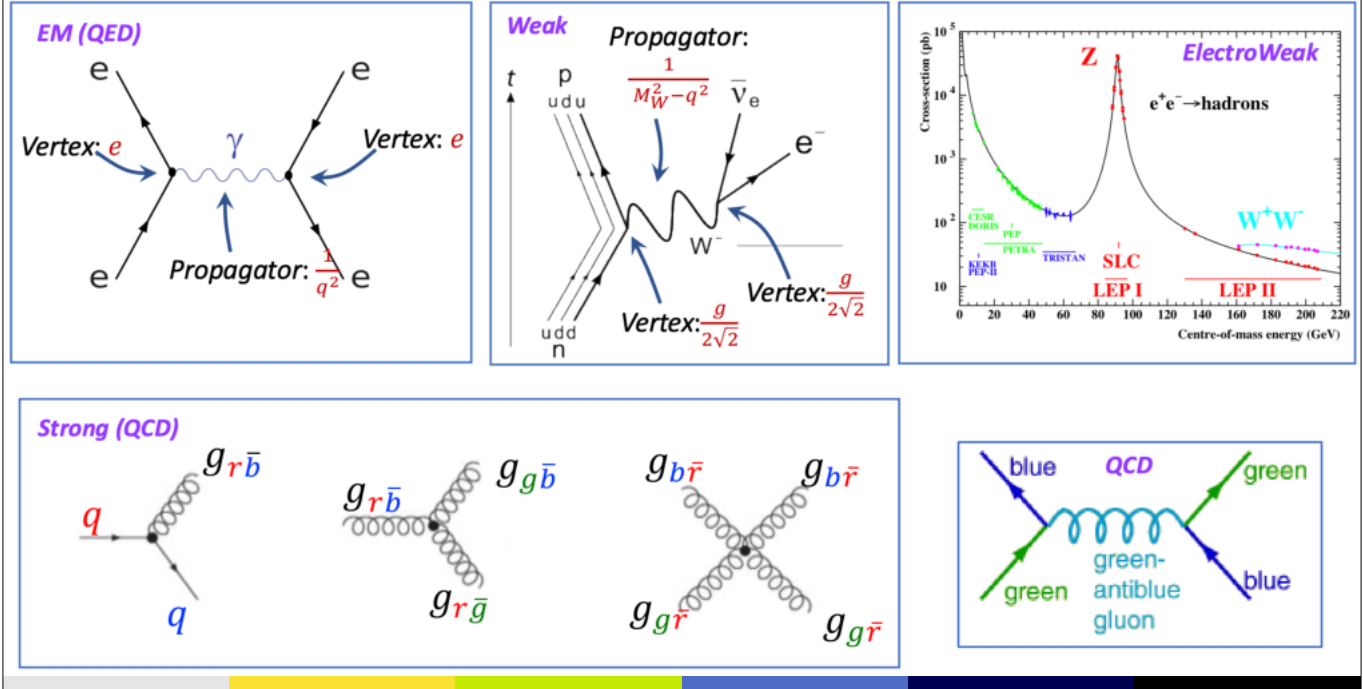
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Classification of particles

- Lepton**: fundamental particle
- Hadron**: consist of **quarks**
 - Meson**: 1 quark + 1 antiquark (π^+ , B_s^0 , ...)
 - Baryon**: 3 quarks (p , n , Λ , ...)
 - Anti-baryon**: 3 anti-quarks
- Fermion**: particle with half-integer spin.
 - Antisymmetric wave function: obeys Pauli-exclusion principle and Pauli-Dirac statistics
 - All fundamental quarks and leptons are spin- $1/2$
 - Baryons ($S=1/2, 3/2$)
- Boson**: particle with integer spin
 - Symmetric wave function: Bose-Einstein statistics
 - Mesons**: ($S=0, 1$), **Higgs** ($S=0$)
 - Force carriers**: γ, W, Z, g ($S=1$); graviton ($S=2$)



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- The Lagrangian of the Standard Model includes electromagnetic, weak and strong interactions according to the gauge field principle

- Construction of the Lagrangian: $\mathcal{L} = \mathcal{L}_{\text{free}} - \mathcal{L}_{\text{interaction}} = \mathcal{L}_{\text{Dirac}} - g J^\mu A_\mu$
 - With g a coupling constant, J^μ a current ($\bar{\psi} O_i \psi$) and A_μ a force field

A. Local $U(1)$ gauge invariance: symmetry under complex phase rotations

- Conserved quantum number: (hyper-) charge

- Lagrangian: $\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - q \underbrace{\bar{\psi}\gamma^\mu \psi}_{J_{EM}^\mu} A_\mu$
 $(\partial_\mu \rightarrow D_\mu \equiv \partial_\mu + iqA_\mu)$

Note Spinor: $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$

B. Local $SU(2)$ gauge invariance: symmetry under transformations in isospin doublet space.

- Conserved quantum number: weak isospin

- Lagrangian: $\mathcal{L} = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi = \bar{\Psi}(i\gamma^\mu \partial_\mu - m)\Psi - \frac{g}{2} \underbrace{\bar{\Psi}\gamma^\mu \vec{\tau}\Psi}_{J_{Weak}^\mu} \vec{b}_\mu$
 $(i\partial_\mu \rightarrow D_\mu = i\partial_\mu + igB_\mu) \quad ; \quad B_\mu = \frac{1}{2} \vec{\tau} \cdot \vec{b}_\mu = \frac{1}{2} \tau^a b_\mu^a = \frac{1}{2} \begin{pmatrix} b_3 & b_1 - ib_2 \\ b_1 + ib_2 & -b_3 \end{pmatrix}$

Note doublet spinors: $\Psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$
 with ψ_u, ψ_d spinors

C. Local $SU(3)$ gauge invariance: symmetry under transformations in colour triplet space

- Conserved quantum number: color

- Lagrangian: $\mathcal{L} = \bar{\Phi}(i\gamma^\mu D_\mu - m)\Phi = \bar{\Phi}(i\gamma^\mu \partial_\mu - m)\Phi - \frac{g_s}{2} \underbrace{\bar{\Phi}\gamma^\mu \vec{\lambda}\Phi}_{J_{QCD}^\mu} \vec{c}_\mu$
 $(i\partial_\mu \rightarrow D_\mu = i\partial_\mu + ig_s C_\mu) \quad C_\mu \text{ are } 3 \times 3 \text{ matrices} \rightarrow \text{gluon fields}$

Note triplet spinors: $\Phi = \begin{pmatrix} \psi_r \\ \psi_g \\ \psi_b \end{pmatrix}$
 ψ_r, ψ_g, ψ_b spinors

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- The Lagrangian of the Standard Model includes electromagnetic, weak and strong interactions according to the gauge field principle
- Construction of the Lagrangian: $\mathcal{L} = \mathcal{L}_{\text{free}} - \mathcal{L}_{\text{interaction}} = \mathcal{L}_{\text{Dirac}} - gJ^\mu A_\mu$
 - With g a coupling constant, J^μ a current ($\bar{\psi}O_i\psi$) and A_μ a force field

Standard Model Lagrangian:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi - qJ_{EM}^\mu A_\mu - \frac{g}{2}\vec{J}^\mu_{\text{Weak}} \cdot \vec{b}_\mu - \frac{g_s}{2}\vec{J}^\mu_{\text{QCD}} \cdot \vec{c}_\mu$$

Implements U(1), SU(2) and SU(3) symmetries simultaneous:

$$SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y$$

Requiring the Lagrangian to be invariant (symmetry) implies that the EM, Weak and Strong force fields must exist and the interactions respectively conserve charge, weak isospin, and color.

- Massive particles are forbidden in the SM Lagrangian
 - A hypothetical mass term in the Lagrangian for the photon is not gauge invariant under $A^\mu \rightarrow A^{\mu'}$:

$$m^2 A_\mu A^\mu \rightarrow m^2 \left(A_\mu + \frac{1}{e} \partial_\mu \alpha \right) \left(A^\mu + \frac{1}{e} \partial^\mu \alpha \right) \neq m^2 A_\mu A^\mu$$
- The same holds (harder to show) for the weak mediators W, Z
 - However they **are** massive
 - \rightarrow SU(2)xU(1) symmetry is *broken*

The Higgs mechanism breaks the symmetry of the (electro-)weak interaction

- Works along the lines as described in previous slides; introduce a complex SU(2) doublet

• Electroweak Lagrangian: $\mathcal{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi + (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi)$

• Where the covariant derivatives:

U(1): $\psi(x) \rightarrow \psi'(x) = e^{i\alpha(x)}\psi(x)$

and SU(2): $\psi(x) \rightarrow \psi'(x) = G(x)\psi(x)$

$$A^\mu(x) \rightarrow A'^\mu(x) = A^\mu(x) - \frac{1}{q} \partial^\mu \alpha(x)$$

$$\Rightarrow D^\mu = \partial^\mu + iqA^\mu$$

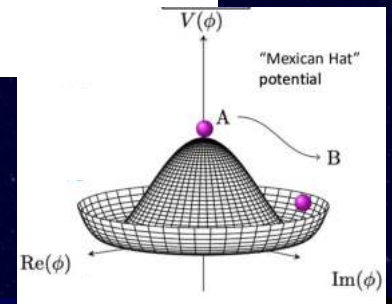
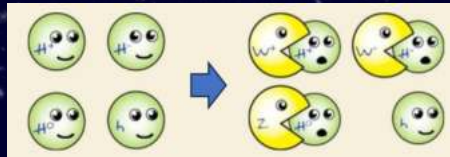
with $G(x) = \exp\left(\frac{i}{2} \vec{\tau} \cdot \vec{\alpha}(x)\right)$

$$B'_\mu = GB_\mu G^{-1} + \frac{i}{g} (\partial_\mu G) G^{-1}$$

$$\Rightarrow D_\mu = I\partial_\mu + igB_\mu$$

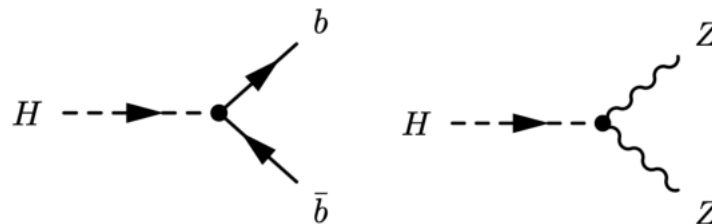
• Higgs field is weak isospin doublet: $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$; $\phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

• With the potential: $V(\phi) = \mu^2(\phi^\dagger \phi) + \lambda(\phi^\dagger \phi)^2$ where: $\mu^2 < 0$

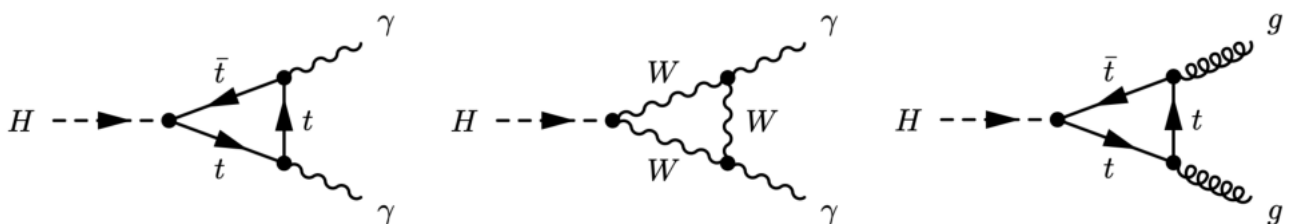


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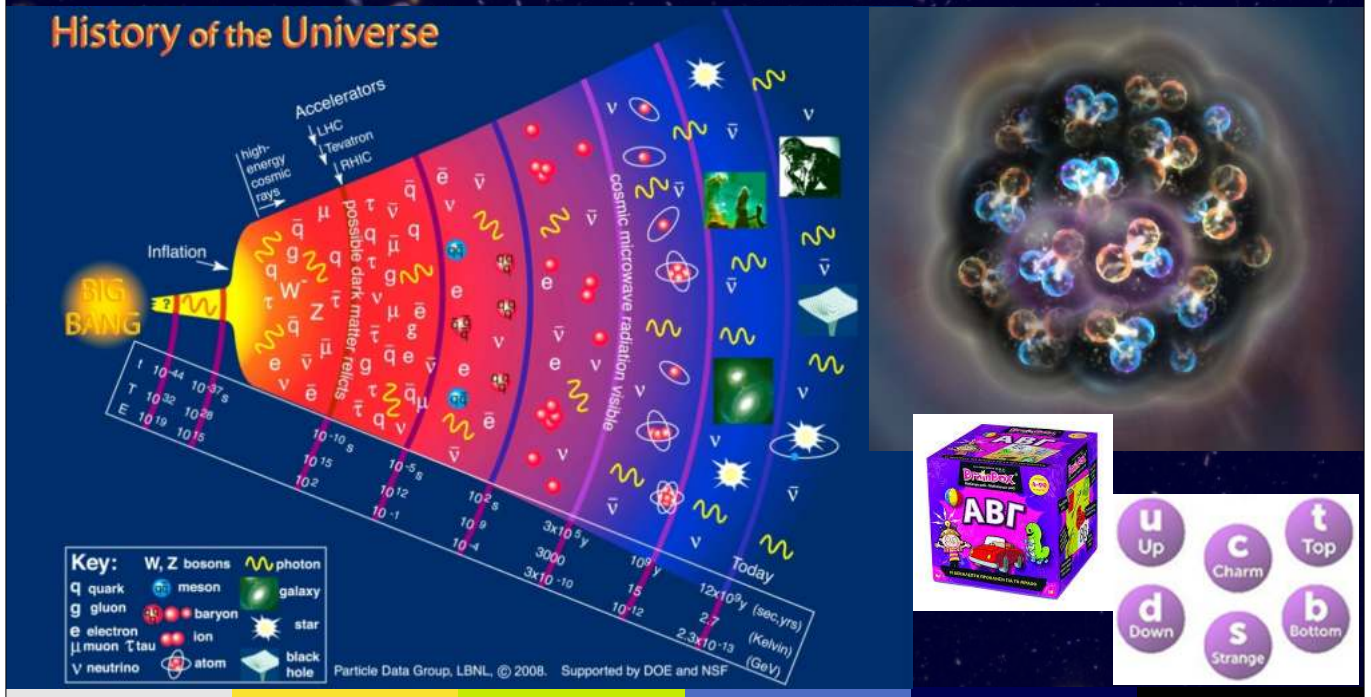
Directly to two fundamental fermions or bosons, coupling to mass, e.g.



Indirectly to massless particles (photons or gluons) via massive loops



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get this lecture



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