Αξιόνια, Κοσμικός Πληθωρισμός και Κβαντική βαρύτητα

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OUTLINE

PART I

- General Relativity as an Effective Field Theory

- Quantum Formalism for weak gravity
  EFT 1-loop action quadratic in gravitons
  - quantum correction in Newtonian potential

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PART II – OTHER APPLICATIONS

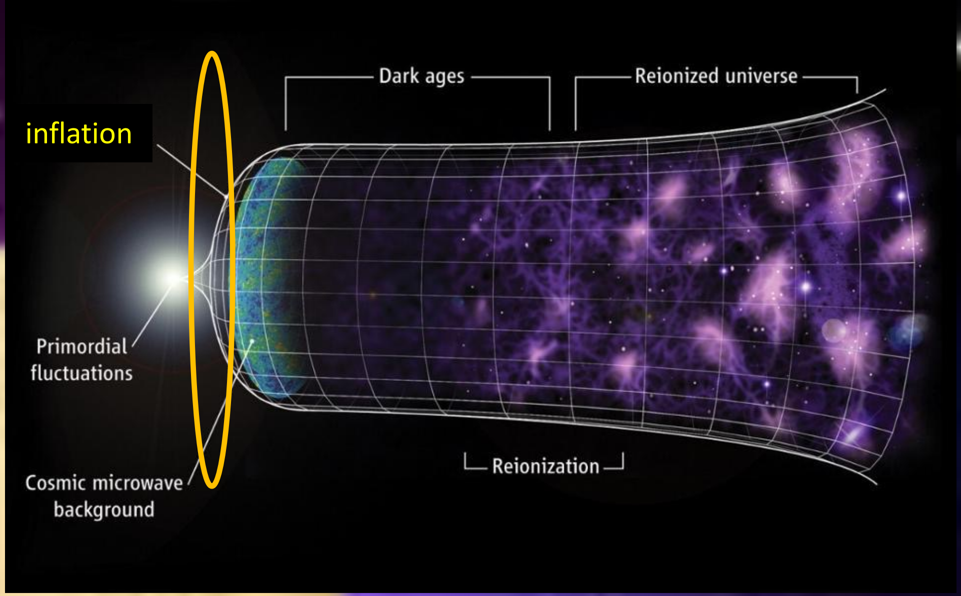
Weak Gravity Canonical Quantization

- The role of axions & gravitational Chern-Simons (gCS) anomalies as string-inspired gravitational EFT embedded in UV complete string models
- gCS Condensate-induced Inflation (String-inspired Running-Vacuum type)
- Squeezed graviton states from Astrophysical rotating Black Holes: a novel (QUANTUM-OPTICS inspired) route for observing quantum signatures in gravitational waves

Advanced topics

- A FUTURE LOOK

OUTLINE



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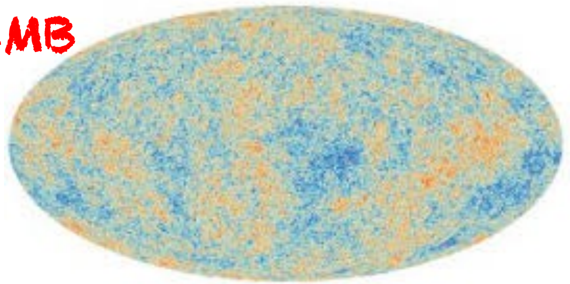
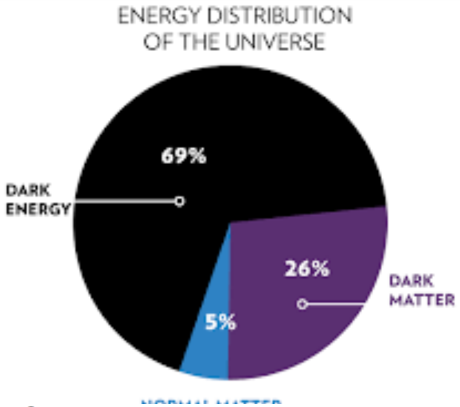
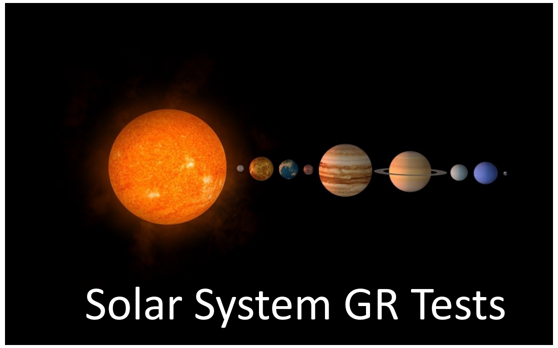
Important (> 26 yrs) Discoveries in Cosmology/Astronomy/Local ((solar system) measurments

CMB

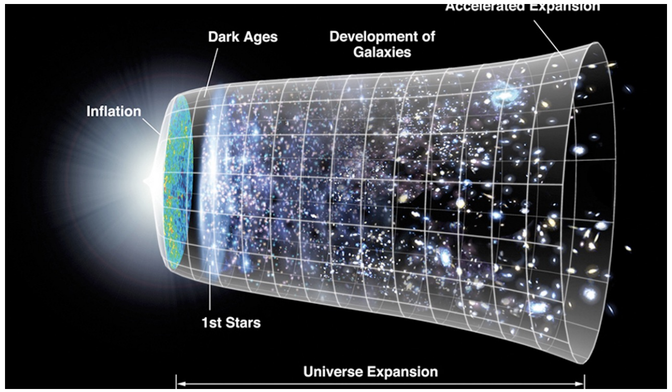


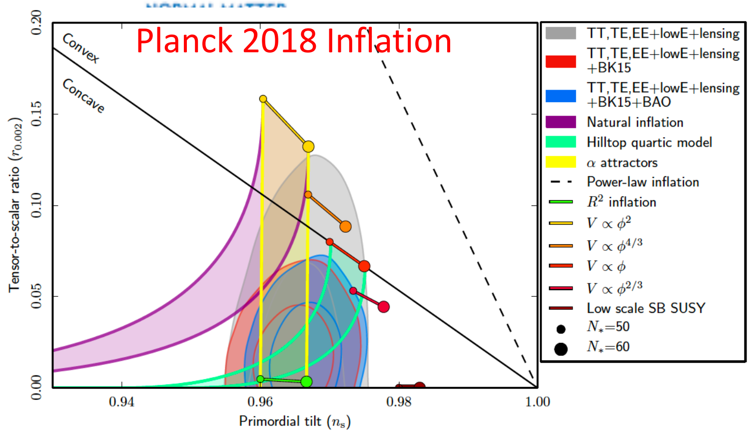




+ SnIa

Helped towards better understanding of evolution of Universe, showed current acceleration ← cosmol. constant (?) dominance



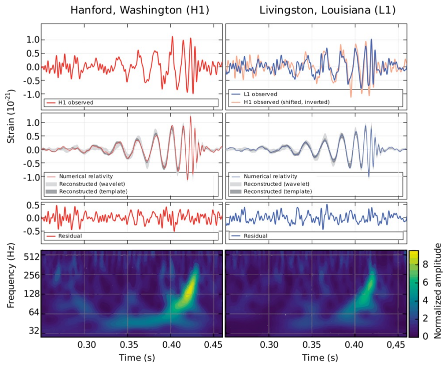


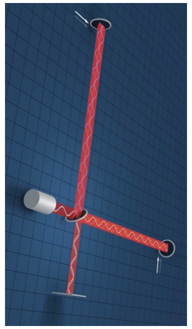
Cosmic time →

Inflation (de Sitter) → radiation-dominance → matter dominance → de Sitter (?) again

Gravitational Waves from Black Hole mergers

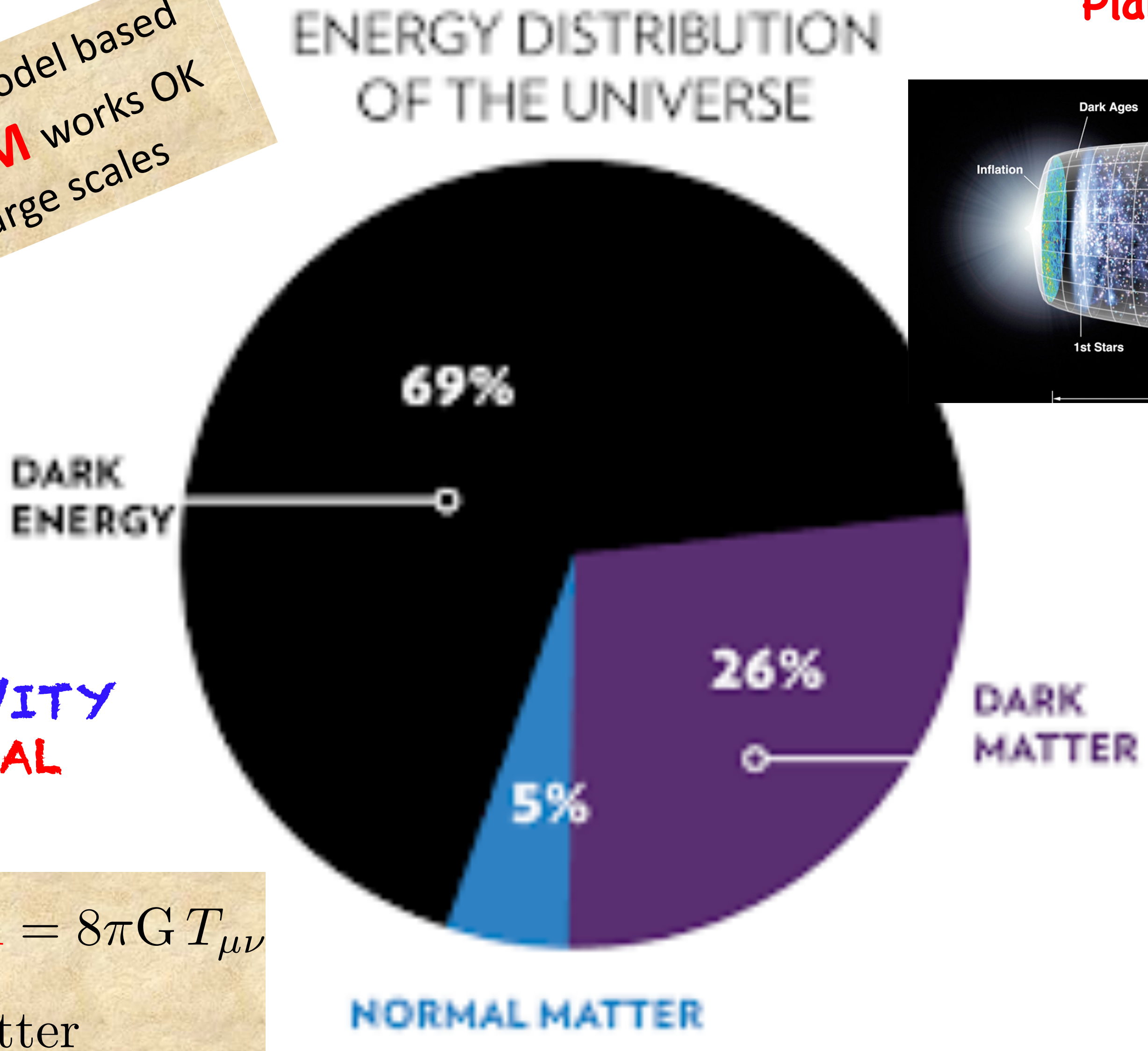
``Heard'' (2015) for the first time by LIGO Inteferometer Open new era in Astronomy

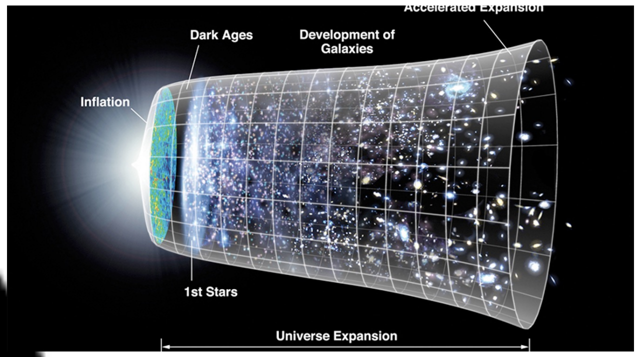




Simplest model based on **ΛCDM** works OK for large scales

Planck2018 data





ALL CONFIRM GENERAL RELATIVITY (GR) AS A CLASSICAL FIELD THEORY

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\, R - g_{\mu\nu}\, \Lambda = 8\pi G\, T_{\mu\nu}$$

$$T_{\mu\nu} \ni \text{Cold Dark Matter}$$

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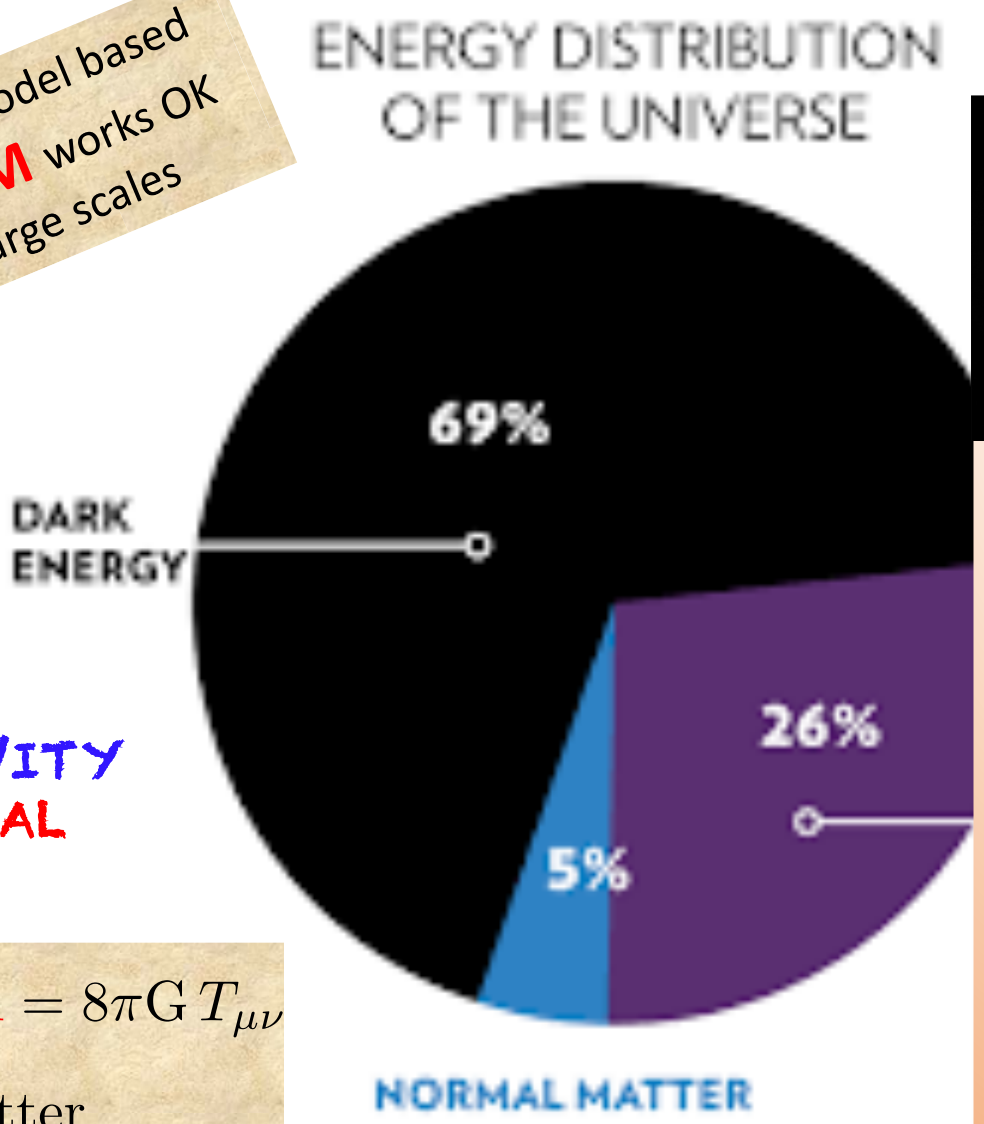

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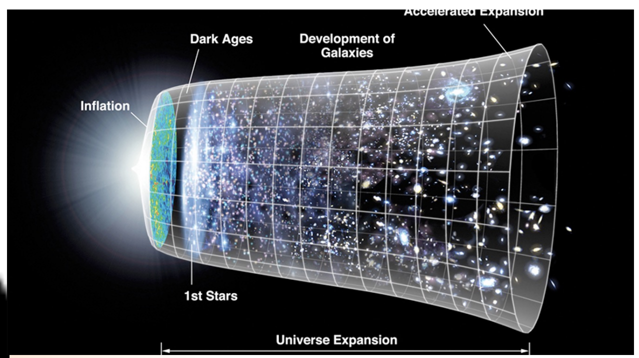

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But: Is the gravitational force a fundamental interaction? or emergent (e.g. entropic - emergent due to quantum entanglement of bits of space-time information, i.e. statistical info @ quantum level encoded in position of material bodies) (Verlinde) ?

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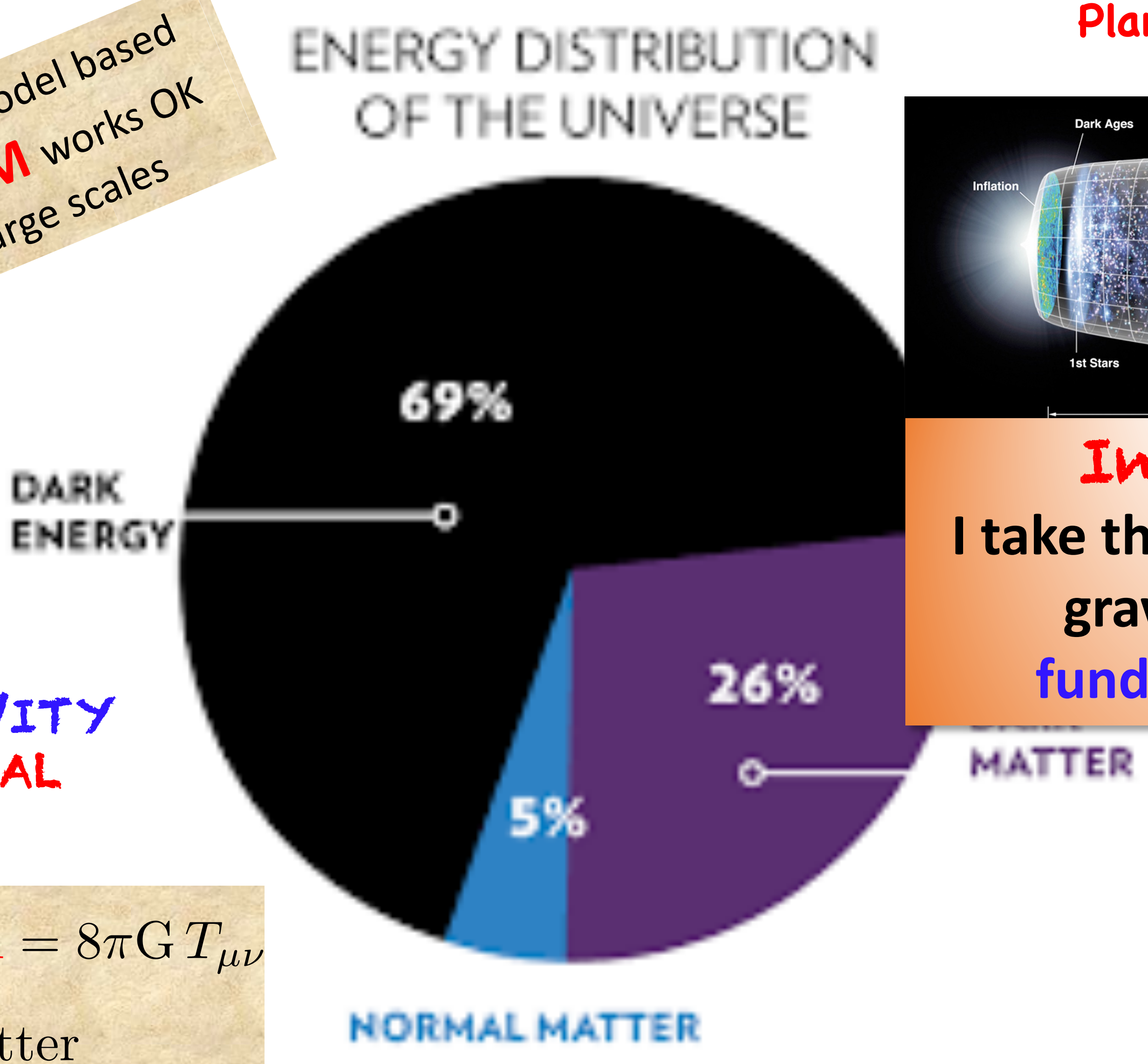

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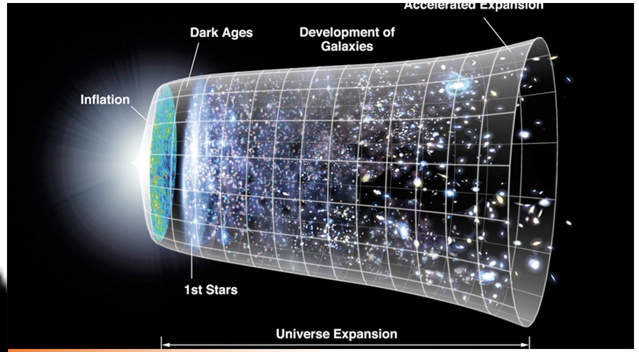

In this lecture:

I take the point of view that the gravitational force is a **fundamental interaction**

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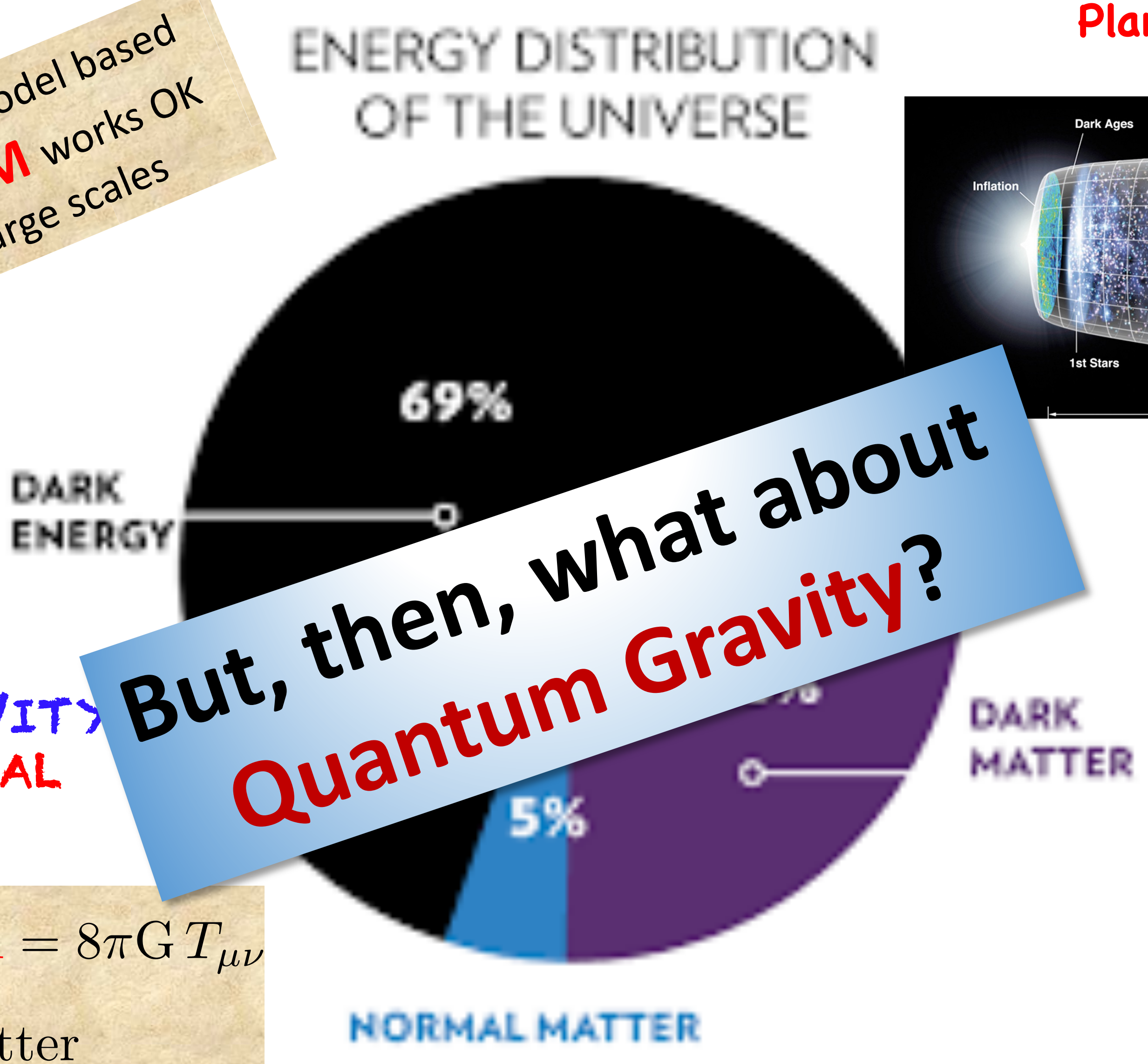

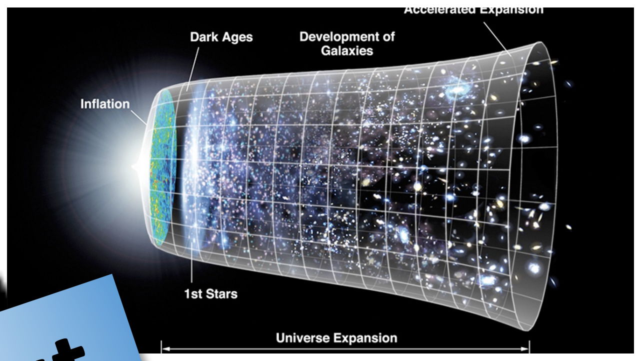

But, then, what about Quantum Gravity?

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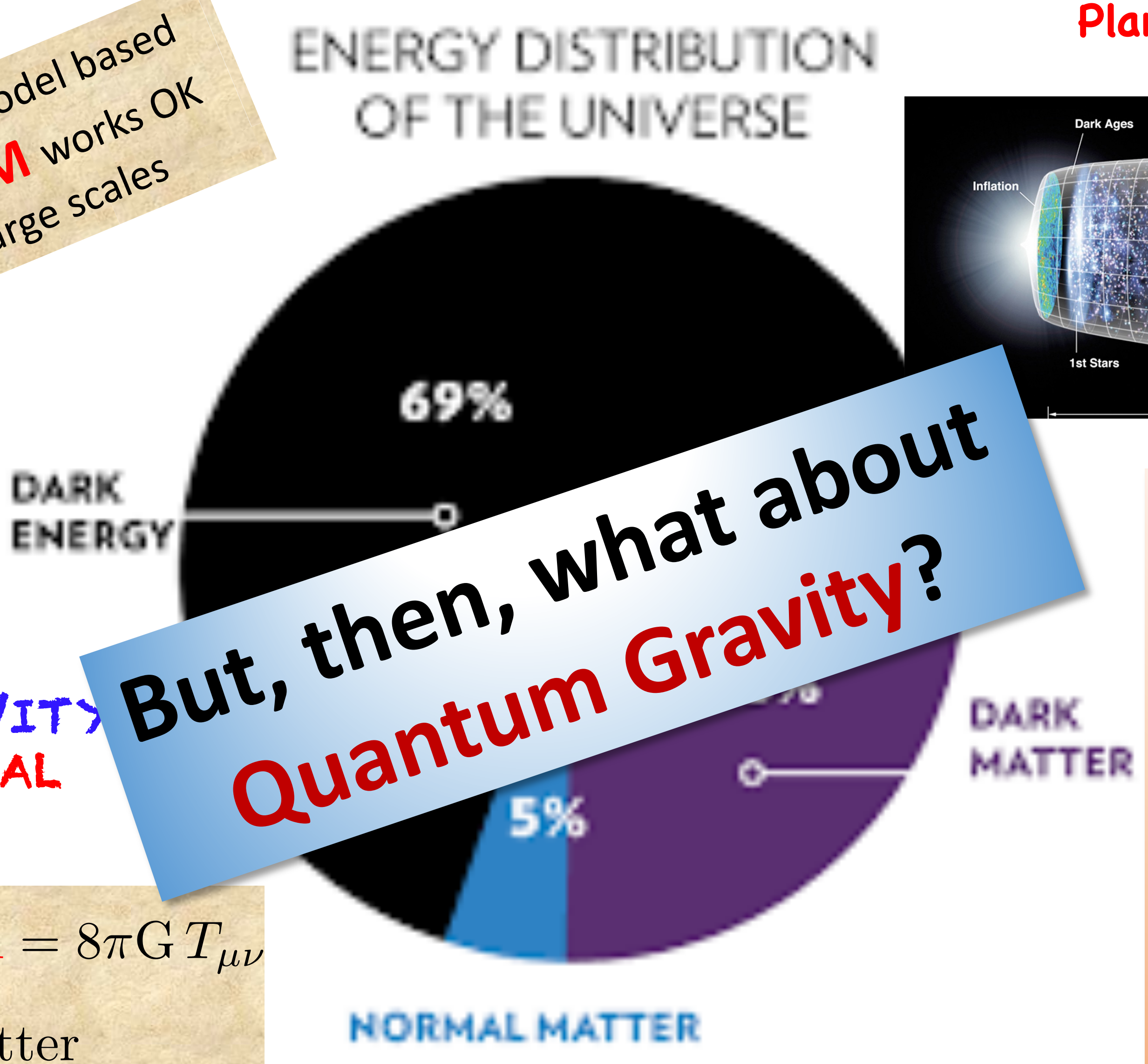

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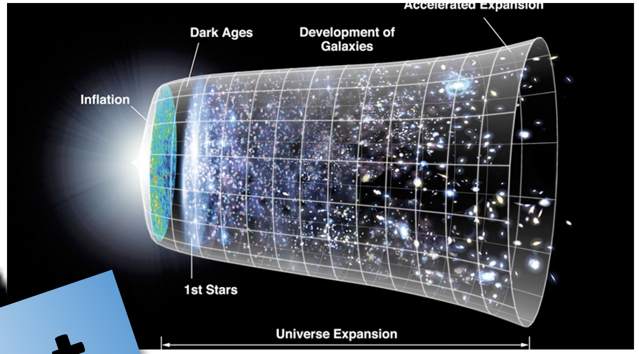

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Can gravity be Quantised as a field theory similarly to the rest of fundamental Interactions in Nature?

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GR as an Effective Field Theory (EFT)

Low energy degrees of freedom organize themselves as quantum graviton fields, $h_{\mu\nu}$, governed by a local Lagrangian, in general containing non-renormalizable terms suppressed by powers of a heavy scale. Nevertheless, **one can make predictions without knowledge of the full high energy Quantum Gravity theory**.

~~UV(unknown)~~ → IR(known)

Full Quantum Gravity

GR EFT
(*cf.* asymptotic safety etc)

G. 't Hooft & M. Veltman, Ann. Inst. Henri Poincaré A20 (1974) , 69

J.F. Donoghue, PRD50 (1994), 3874, 9512024 [gr-qc] 2211.09902 [hep-th]

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Basic approximation:
Weak Gravitational interaction
(Perturbative - infrared (large scales)

G. 't Hooft & M. Veltman, Ann. Inst. Henri Poincaré A20 (1974) , 69

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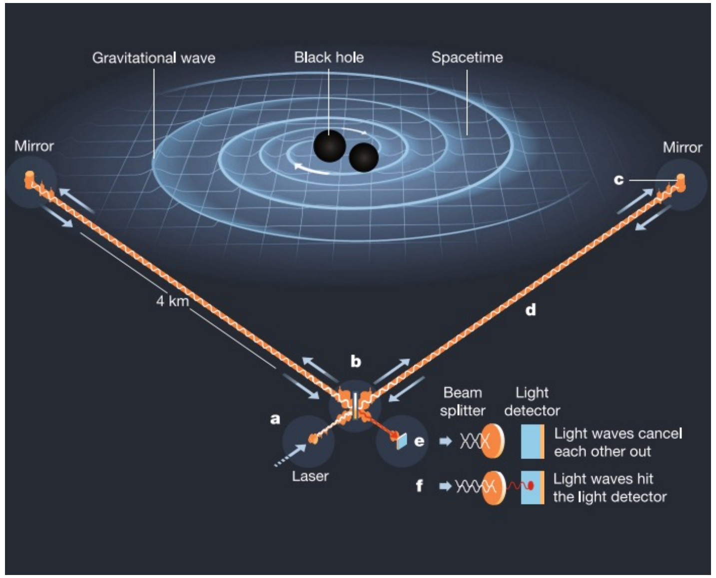

Enforce EFT to study the quantum nature of GWs

$$\boxed{g_{\mu\nu} = g^{(0)}_{\mu\nu} + \kappa\, h_{\mu\nu}}\,, \quad |\kappa h_{\mu\nu}| \ll 1$$

background (for $g^{(0)}_{\mu\nu}$): satisfies classical gravitational eqs

graviton excitation (quantized) (for $h_{\mu\nu}$)

$$\boxed{\kappa = 2\sqrt{8\pi G} = 2\, M_{\rm Pl}^{-1}}$$

$$G^{-1} \simeq (1.2 \times 10^{19}\ {\rm GeV})^2$$

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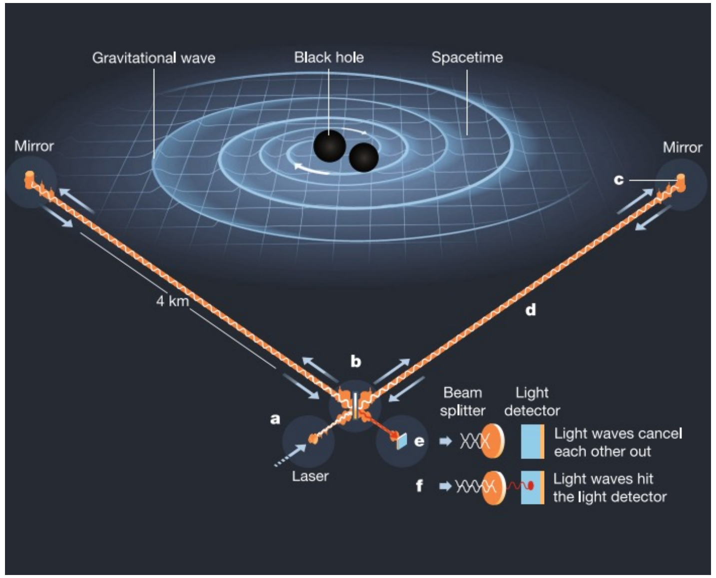

Enforce EFT to study the quantum nature of GWs

Can **test predictions** of weak gravity **EFT,** e.g. if (observed) **signal** is **too strong** → **deviation from EFT**

$$g_{\mu\nu} = g_{\mu \dots} \quad 1$$

backgro...

satisfies classic... gravitational eqs

$$= 2\sqrt{8\pi G}$$
$$= 2\,M_{\rm Pl}^{-1}$$

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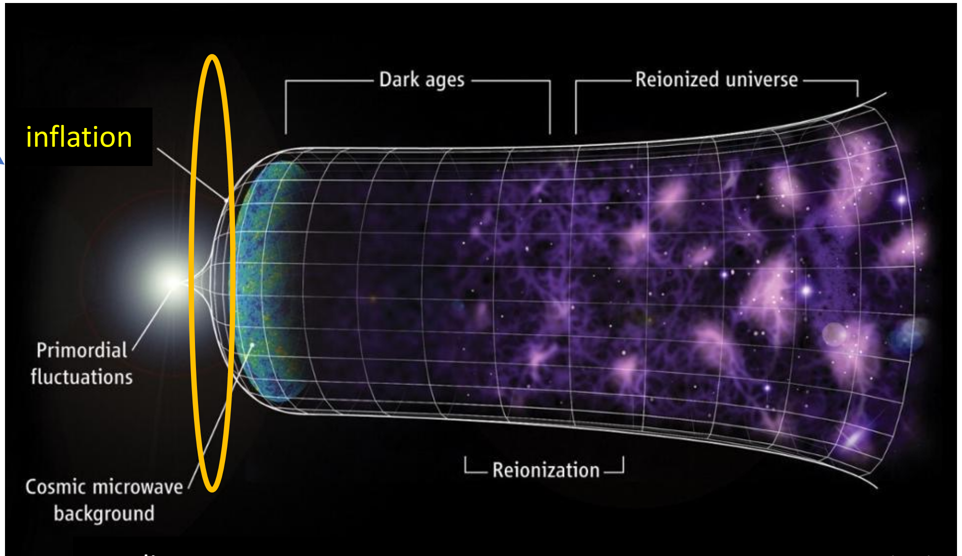

credit C. Faucher-Giguère, A. Lidz, L. Hernquist, Science 319, 5859 (47)

Enforce EFT to study the potential quantum nature of Inflation ?

Part I

GR - conventions of

J.F. Donoghue, PRD50 (1994), 3874, 9512024 [gr-qc]

$$\boxed{S_{\mathbf{grav}} = \int d^4x\sqrt{-g}\left[\frac{2}{\kappa^2}R\right]}$$

$$R_{\mu\nu} = \partial_\nu\Gamma^\lambda_{\mu\lambda} - \partial_\lambda\Gamma^\lambda_{\mu\nu} + \Gamma^\sigma_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\sigma_{\mu\nu}\Gamma^\lambda_{\lambda\sigma}$$

$$\Gamma^\lambda_{\alpha\beta} = \frac{g^{\lambda\sigma}}{2}\left(\partial_\alpha g_{\beta\sigma} + \partial_\beta g_{\alpha\sigma} - \partial_\sigma g_{\alpha\beta}\right) .$$

$$\kappa^2 = 32\pi G, g = \det g_{\mu\nu};$$

$$R = g^{\mu\nu}R_{\mu\nu}$$

Metric signature: (+ , -, -, -) (``particle physics (Dirac, S. Weinberg)'')

Indicative Example of matter : (massive) scalar fields interacting with gravity

$$\boxed{S_{\mathbf{matter}} = \int d^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}m^2\phi^2\right]}$$

$$\boxed{S_{\rm total} = S_{\rm grav} + S_{\rm matter}}$$

Weak Quantum-Graviton Fluctuations :

Spacetime-Background-dependent approach

J.F. Donoghue, PRD50 (1994), 3874, 9512024 [gr-qc]

$$\boxed{g_{\mu\nu} = g^{(0)}_{\mu\nu} + \kappa\, h_{\mu\nu}}\;, \qquad |\kappa h_{\mu\nu}| \ll 1$$

Notation : $g^{(0)}_{\mu\nu} \;\rightarrow\; \overline{g}_{\mu\nu}$

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satisfies classical gravitational eqs

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NB: contrast with UV complete full (strongly coupled, non-perturbative) quantum gravity (QG) theory (still unknown) which is supposed to be background independent → generate dynamically spacetime → includes at low energies, weak gravity limit GR as EFT

Various approaches of UV complete QG :

(i) background independent : loop QG, spin-foam models, group-field theory

(ii) (so dfar) Background dependent: string theory . . .

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$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu}\;, \quad \textbf{(exact)}$$

$$g^{\mu\nu} = \bar{g}^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu}_{\lambda} h^{\lambda\nu} + \cdots$$

Sufficient approximation for weak QG effective action:

quadratic terms in graviton excitations $h_{\mu\nu}$ & *@ one loop*

- matter interactions: (massive) scalar fields

G. 't Hooft & M. Veltman, **Ann. Inst. Henri Poincaré A20 (1974) , 69**

Weak QG effective action :
quadratic terms in graviton excitations $h_{\mu\nu}$ &
@ one loop:
-Pure gravity: all UV divergencies absorbed in field (graviton) normalization
- Matter interactions: (massive) scalar fields
UV Divergencies in physical Quantities remain

G. 't Hooft & M. Veltman,
Ann. Inst. Henri Poincaré A20 (1974) , 69

Weak Gravity EFT as a Gauge Theory

GR Lagrangian is invariant under gauge transformations
- general coordinate (diffeomorphism) invariance
→ redundant d.o.f. gravitational field
→ need for gauge fixing

→ Infinitesimal diffeomorphisms

$$\boxed{\begin{aligned} x^\mu &\to x^\mu + \epsilon^\mu(x), \\ g_{\mu\nu} &\to g_{\mu\nu} - \mathcal{L}_\xi g_{\mu\nu} \\ &= g_{\mu\nu} - \epsilon_{\mu\,;\,\nu} - \epsilon_{\nu\,;\,\mu} \end{aligned}}$$

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Differences from ordinary (e.g Yang-Mills (YM)) gauge theories:

In YM : gauge trnsf → vertical bundle automorphisms acting only on the ``internal space''

In Gravity: general coordinate (diffeomorphisms) are active trnsf. changing spacetime points & acting on th tangent space of a manifold at a point

→ passive point of view: generated by Lie derivative on metric $\mathcal{L}_\xi g_{\mu\nu}$

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C

Ordinary Non-Abelian
Gauge Theories

NB

Gauge fixing in (non-Abelian) gauge theories → Fadeev – Popov ghosts

Gauge trnsf: $\mathbf{A} \equiv A^a_\mu \mathbf{T}^a = \mathbf{U}\,\bar{\mathbf{A}}\,\mathbf{U}^\dagger + \frac{i}{g}(\partial_\mu \mathbf{U})\,\mathbf{U}^\dagger \simeq \bar{\mathbf{A}}_\mu + \frac{1}{g}\bar{\mathbf{D}}_\mu \theta\,,$

$$\mathbf{D}_\mu \theta \equiv (D_\mu \theta)^a\,\mathbf{T}^a\,, \quad (D_\mu\theta)^a = \partial_\mu \theta^a + g\,f^{abc}\,A^b_\mu\,\theta^c$$

, $|\theta^a| \ll 1$

infinitesimal

Gauge fixing (e.g. Lorentz gauge) $\partial_\mu \mathbf{A}^\mu = 0 \stackrel{\text{gauge trnsf}}{\longrightarrow} \partial_\mu \bar{\mathbf{A}}^\mu + \partial_\mu(\bar{\mathbf{D}}^\mu \theta) \equiv G(\bar{A}^a_\mu, \theta^b) = 0$

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x , y

Introduce **this constraint** In the **path integral** via **δ-functional**

$$Z = \int dy \int dx\,\delta(y - f(x))\,e^{iS[x]}$$

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, $|\theta^a| << 1$

infinitesimal

Gauge fixing

(e.g. Lorentz gauge) $\partial_\mu \mathbf{A}^\mu = 0 \overset{\text{gauge trnsf}}{\longrightarrow}$ $\boxed{\partial_\mu \bar{\mathbf{A}}^\mu + \partial_\mu(\bar{\mathbf{D}}^\mu\theta) \equiv G(\bar{A}^a_\mu, \theta^b) = 0}$

x , y

Introduce **this constraint** In the **path integral** via **δ-functional**

$$Z = \int dy \int dx\, \delta(y - f(x))\, e^{iS[x]} \;\rightarrow\; \delta(y - f(x)) = \delta(G(y,x))\,(\frac{\partial G}{\partial y}) \;\rightarrow$$ **assume** $\frac{\partial G}{\partial y} > 0,$

$$\rightarrow \boxed{Z = \int dy \int dx\, \frac{\partial G}{\partial y}\,\delta(G(y,x))\, e^{iS[x]}}$$

NB

Gauge fixing in (non-Abelian) gauge theories → Fadeev – Popov ghosts

Gauge trnsf: $\mathbf{A} \equiv A^a_\mu \mathbf{T}^a = \mathbf{U}\,\bar{\mathbf{A}}\,\mathbf{U}^\dagger + \frac{i}{g}(\partial_\mu \mathbf{U})\,\mathbf{U}^\dagger \simeq \bar{\mathbf{A}}_\mu + \frac{1}{g}\bar{\mathbf{D}}_\mu \theta\,,$

$$\mathbf{D}_\mu \theta \equiv (D_\mu \theta)^a\, \mathbf{T}^a\,, \qquad (D_\mu \theta)^a = \partial_\mu \theta^a + g\, f^{abc}\, A^b_\mu\, \theta^c$$

, $|\theta^a| \ll 1$ infinitesimal

Gauge fixing (e.g. Lorentz gauge) $\partial_\mu \mathbf{A}^\mu = 0 \xrightarrow{\text{gauge trnsf}} \boxed{\partial_\mu \bar{\mathbf{A}}^\mu + \partial_\mu (\bar{\mathbf{D}}^\mu \theta) \equiv G(\bar{A}^a_\mu, \theta^b) = 0}$

In the non-Abelian Gauge case

$$G^a(x) = \partial^\mu \bar{A}^a_\mu(x) + \omega^a(x), \quad a = 1, \ldots, \dim[\mathrm{G}]$$

$$\boxed{Z = \int d\theta \int dA\, \frac{\partial G}{\partial \theta}\, \delta(G(\theta, A))\, e^{iS[A]}}$$

$$\frac{\delta G^a(x)}{\delta \theta^b(x')} = \partial^\mu D^a_{\mu\, b}\, \delta^{(4)}(x - x')$$

NB

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Determinant

NB Gauge fixing in (non-Abelian) gauge theories → Fadeev – Popov ghosts

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$$\boxed{Z = \int d\theta \int dA \frac{\partial G}{\partial \theta} \delta(G(\theta, A))\, e^{iS[A]}}$$

Δ_{FP}=Det$[\,\partial^\mu D^a_{\mu\,b}\,\delta^{(4)}(x - x')]$

Fadeev-Popov Determinant

NB

Gauge fixing in (non-Abelian) gauge theories → Fadeev – Popov ghosts

Represent FP determinant using Grassmann fields

$$\boxed{\Delta_{FP} = \mathrm{Det}\left[\frac{\delta G^a(x)}{\delta\theta^b(x')}\right] \propto \int D\bar{c}\, Dc\, \exp\left(i\int d^4x\, \mathcal{L}_{\mathrm{FP}}\right)}$$

$$\int d^4x\, \mathcal{L}_{\mathrm{FP}} = \int d^4x\, \bar{c}_a\, \partial^\mu\, D_{\mu\ \ b}^{a}\, c^b = -\int d^4x\, \partial^\mu\, \bar{c}_a\, D_{\mu\ \ b}^{a}\, c^b$$

$$= -\int d^4x\, \partial^\mu\, \bar{c}_a\, \partial_\mu\, c^a + \int d^4x\, g\, f^{abc}\, \partial^\mu \bar{c}_a\, A_\mu^c\, c^b$$

NB

Gauge fixing in (non-Abelian) gauge theories → Fadeev – Popov ghosts

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⚠

wrong sign kinetic term
→ Fadeev Popov Ghosts

NB Using $G^a(x) = \partial^\mu A^a_\mu(x) + \omega^a(x), \quad a = 1, \ldots, \dim[\mathrm{G}]$

and gauge invariance of the YM action S_{YM} and Z

$$Z = \int d\theta \int dA \, \frac{\partial G}{\partial \theta} \, \delta(G(\theta, A)) \, e^{iS[A]} \times f(\omega^\alpha)$$

any function of ω^α will not affect Z due to Gauge Invariance

NB Using $G^a(x) = \partial^\mu A^a_\mu(x) + \omega^a(x), \quad a = 1,\ldots,\dim[\mathrm{G}]$

and gauge invariance of the YM action S_{YM} and Z

$$Z = \int d\theta \int dA \, \frac{\partial G}{\partial \theta} \, \delta(G(\theta, A)) \, e^{iS[A]} \times \exp\left(-\frac{i}{2\xi} \int d^4x \, \omega^a(x) \, \omega^a(x)\right)$$

can chose this form

$$\boxed{Z[J] \propto \int DA \, D\bar{c} \, Dc \, \exp\left(iS_{\mathrm{YM}} + i \int d^4x \mathcal{L}_{\mathrm{FP}} - i \int d^4x \, \frac{1}{2\,\xi} \, \partial^\mu A^a_\mu \, \partial^\nu A^a_\nu\right)}$$

R_ξ gauge -fixing term

→ ghost + gauge-fixing sectors exhibit residual quantum symmetry: BRST symmetry (Becci, Rouet, Stora, Tyupkin)

)

Back to (weak) Gravity as a Gauge Theory

Weak field Gravity as a (one-loop) Gauge EFT

Warming up: Classical considerations

G. 't Hooft & M. Veltman, Ann. Inst. Henri Poincaré A20 (1974) , 69

J.F. Donoghue, PRD50 (1994), 3874, 9512024 [gr-qc]

Background spacetime : Minkowski

$$R_{\mu\nu} = \frac{\kappa}{2}\left[\partial_\mu\partial_\nu h^\lambda{}_\lambda + \partial_\lambda\partial^\lambda h_{\mu\nu} - \partial_\mu\partial_\lambda h^\lambda{}_\nu - \partial_\lambda\partial_\nu h^\lambda{}_\mu\right] + \mathcal{O}(h^2)$$

$$R = \kappa\left[\Box h^\lambda{}_\lambda - \partial_\mu\partial_\nu h^{\mu\nu}\right] + \mathcal{O}(h^2)$$

gauge fixing
harmonic gauge:

$$\partial^\lambda h_{\mu\lambda} = \frac{1}{2}\partial_\mu h^\lambda{}_\lambda$$

Notation from now on:

$$h \equiv h^\lambda{}_\lambda$$

Einstein's equations for linearized gravity

$$\Box h_{\mu\nu} = -16\pi G\left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}T^\lambda{}_\lambda\right)$$

Solution for a static point-like mass M $\quad h_{\mu\nu} = diag(1,1,1,1)\left(2\frac{GM}{r}\right)$

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\begin{aligned}
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R &= \kappa\left[\Box h^\lambda{}_\lambda - \partial_\mu\partial_\nu h^{\mu\nu}\right] + \mathcal{O}(h^2)
\end{aligned}
$$

gauge fixing
harmonic gauge:

$$\partial^\lambda h_{\mu\lambda} = \frac{1}{2}\partial_\mu h^\lambda{}_\lambda$$

Notation from now on:

$$h \equiv h^\lambda{}_\lambda$$

Einstein's equations for linearized gravity

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$$h_{\mu\nu} = N\epsilon_{\mu\nu}e^{-ip\cdot x} + h.c.$$

graviton dispersion
$p^2 = 0$
massless

Plane-gravitational-wave solutions $R_{\mu\nu} = 0 = \Box h_{\mu\nu}$

Higher-order gravity → gravitational-wave (GW) energy momentum tensor in harmonic gauge

$$
\begin{aligned}
T_{\mu\nu} \;=\; & -\frac{1}{4}h_{\alpha\beta}\partial_\mu\partial_\nu h^{\alpha\beta} + \frac{1}{8}h\partial_\mu\partial_\nu h \\
& +\frac{1}{8}\eta_{\mu\nu}\left(h^{\alpha\beta}\Box h_{\alpha\beta} - \frac{1}{2}h\Box h\right) \\
& -\frac{1}{4}\left(h_{\mu\rho}\Box h^{\rho}{}_{\nu} + h_{\nu\rho}\Box h^{\rho}{}_{\mu} - h_{\mu\nu}\Box h\right) \\
& +\frac{1}{8}\partial_\mu\partial_\nu\left\{h_{\alpha\beta}h^{\alpha\beta} - \frac{1}{2}hh\right\} - \frac{1}{16}\eta_{\mu\nu}\Box\left\{h_{\alpha\beta}h^{\alpha\beta} - \frac{1}{2}hh\right\} \\
& -\frac{1}{4}\partial_\alpha\left[\partial_\nu\left\{h_{\mu\beta}h^{\alpha\beta}\right\} + \partial_\mu\left\{h_{\nu\beta}h^{\alpha\beta}\right\}\right] + \frac{1}{2}\partial_\alpha\left[h^{\alpha\beta}\left(\partial_\nu h_{\mu\beta} + \partial_\mu h_{\nu\beta}\right)\right]
\end{aligned}
$$

Non-linearity of gravity !

Weak-field expansion about an arbitrary spacetime background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu} \ ,$$
$$g^{\mu\nu} = \bar{g}^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu}_{\lambda} h^{\lambda\nu} + \cdots$$

Analogy with non-Abelian gauge theories → gauge-fixing in weak gravity

$$\mathcal{L}_{\rm GF} = \frac{1}{2\xi}(\partial^\mu A^a_\mu)\,(\partial^\nu A^a_\nu) \quad \Rightarrow \quad \boxed{\mathcal{L}_{GF} = \sqrt{-\bar{g}}\left\{\left(h_{\mu\nu}{}^{;\nu} - \frac{1}{2}h_{;\mu}\right)\left(h^{\mu\lambda}{}_{;\lambda} - h^{;\mu}\right)\right\}}$$

Fadeev-Popov ghosts

$$\int d^x\, \mathcal{L}_{\rm FP} = \int d^4x\, \bar{c}_a \partial^\mu\, D^a_{\mu\ b} c^b \quad \Rightarrow \quad \boxed{\mathcal{L}_{\rm ghost} = \sqrt{-\bar{g}}\eta^{*\mu}\left\{\eta_{\mu;\lambda}{}^{;\lambda} - \bar{R}_{\mu\nu}\eta^\nu\right\}}$$

complex Faddeev-Popov ghost field η_μ

Expansion of the Einstein-Hilbert Action in powers of the weak field $h_{\mu\nu}$

$$S_{\mathbf{grav}} = \int d^4x\sqrt{-\bar{g}}\left[\frac{2\bar{R}}{\kappa^2} + \mathcal{L}_g^{(1)} + \mathcal{L}_g^{(2)} + \cdots\right]$$

$$\boxed{\begin{aligned}\mathcal{L}_g^{(1)} &= \frac{h_{\mu\nu}}{\kappa}\left[\bar{g}^{\mu\nu}\bar{R} - 2\bar{R}^{\mu\nu}\right] \;, \\ \mathcal{L}_g^{(2)} &= \frac{1}{2}h_{\mu\nu;\alpha}h^{\mu\nu;\alpha} - \frac{1}{2}h_{;\alpha}h^{;\alpha} + h_{;\alpha}h^{\alpha\beta}{}_{;\beta} - h_{\mu\beta;\alpha}h^{\mu\alpha;\beta} + \bar{R}\left(\frac{1}{4}h^2 - \frac{1}{2}h_{\mu\nu}h^{\mu\nu}\right) + \left(2h^{\lambda}_{\mu}h_{\nu\lambda} - hh_{\mu\nu}\right)\bar{R}^{\mu\nu} \;.\end{aligned}}$$

Expansion of the matter Action in powers of the weak field $h_{\mu\nu}$

$$S_{\text{matter}} = \int d^4x \sqrt{-\bar{g}} \left\{ \mathcal{L}_m^0 + \mathcal{L}_m^{(1)} + \mathcal{L}_m^{(2)} + \cdots \right\}$$

$$\mathcal{L}_m^{(0)} = \frac{1}{2}\left(\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2\right) ,$$

$$\mathcal{L}_m^{(1)} = -\frac{\kappa}{2} h_{\mu\nu} T^{\mu\nu} ,$$

$$T_{\mu\nu} \equiv \partial_\mu \phi \partial_\nu \phi - \frac{1}{2}\bar{g}_{\mu\nu}\left(\partial_\lambda \phi \partial^\lambda \phi - m^2\phi^2\right)$$

$$\mathcal{L}_m^{(2)} = \kappa^2 \left(\frac{1}{2} h^{\mu\nu} h^\nu_\lambda - \frac{1}{4} h h^{\mu\nu}\right) \partial_\mu \phi \partial_\nu \phi - \frac{\kappa^2}{8}\left(h^{\lambda\sigma} h_{\lambda\sigma} - \frac{1}{2} h h\right)\left[\partial_\mu \phi \partial^\mu \phi - m^2 \phi^2\right]$$

Background Einstein Eqs:

$$\bar{R}^{\mu\nu} - \frac{1}{2}\bar{g}^{\mu\nu}\bar{R} = -\frac{\kappa^2}{4} T^{\mu\nu}$$

$$\boxed{\mathcal{L}_g^{(1)} + \mathcal{L}_m^{(1)} \Big|_{\text{Backgr Eqs}} = 0}$$

One-loop weak quantum gravity → restriction to **quadratic order** in graviton fluctuations $h_{\mu\nu}$

$$S_0 = \int d^4x \sqrt{-g} \left\{ \frac{2\bar{R}}{\kappa^2} + \mathcal{L}_m^{(0)} \right\}$$

$$S_{\text{quad}} = \int d^4x \sqrt{-g} \left\{ \mathcal{L}_g^{(2)} + \mathcal{L}_{GF} + \mathcal{L}_{\text{ghost}} + \mathcal{L}_m^{(2)} \right\} .$$

Path-Integral Quantization of weak gravity → generating functionals (formal expressions)

$$Z[\phi] = e^{iW[\phi]} = \int [dh_{\mu\nu}][d\eta_\mu] e^{i(S_0 + S_{\text{quad}})}$$

⇒ **Non-local &** **UV divergent** ⚠

Full: matter + gravity generating functional

$$Z[J] = e^{iW[J]} = \int [d\phi]\, [d\, h_{\mu\nu}]\, e^{iS_{\text{eff}}(\phi, \bar{g}, h, J)}$$

source fields

$$\Delta\mathcal{L} = -J\phi$$

The strategy to deal with this theory

- Dimensionful coupling κ^2 → non-renormalizable + non-linear theory
  → UV cannot be absorbed in existing parameters

- Coupling grows with energy → strongly coupled at Planck energy scales $E \geq M_{Pl}$: perhaps topology changing metric fluctuations
  → full theory of Quantum Gravity (QG) yet unknown
  (but there are candidates: loop QG, spin-foam models, strings)

Low-energy fluctuations much better behave, as they are very weakly coupled

$$\kappa^2 q^2 \in [10^{-40}, \ 10^{-70}] \text{ for } q^2 \in [(\mathrm{fm})^{-2}, \mathrm{m}^{-2}]$$

Naturally separate such low-energy (perturbative) quantum graviton fluctuations from high-energy (non-perturbative) ones within the framework of Effective Field Theory (EFT) of General Relativity (GR) (EFT methods popular in Particle Physics for Beyond Standard Model (BSM) model searches and collider tests)

EFT Methods

Use symmetries of the theory to write all allowed local unitary terms in the effective Lagrangian, compatible with the symmetries, after eliminating non-independent terms, e.g. related through partial integration, mathematical identities *etc*

In **Gravitation**, symmetry is that of invariance **under general coordinate transformations.**

EFT Methods

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In **Gravitation**, symmetry is that of invariance **under general coordinate transformations.**

Derivative expansion is appropriate for separation (decoupling) of high-energy d.o.f. from low-energy d.o.f. **(M_H = high-energy scale)**

e.g. Taylor expansion of propagators, $q^2 << M_H{}^2$

$$\frac{1}{q^2 - M_H^2} = \frac{-1}{M_H^2} - \frac{q^2}{M_H^4} - \frac{q^4}{M_H^6} + \cdots .$$

J.F. Donoghue, PRD50 (1994), 3874, 9512024 [gr-qc]

In pure Gravity we consider Curvature expansion as representing EFT of GR:

Example:

$$S_{\rm grav} = \int d^4x \sqrt{-g} \Big\{ \Lambda + \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \cdots \Big\}$$

Λ = cosmological constant

(NB: In (3+1)-dimensions Gauss-Bonnet topological invariant: $R_{\mu\nu\rho\sigma}\, R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}\, R^{\mu\nu} + R^2 = \mathcal{J}_\mu{}^{;\mu}$)

EFT Methods

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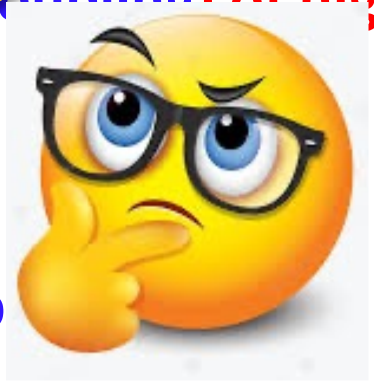
In **Gravitation**, symmetry is th… …ner… …ansformations.

Derivative expansi… …coupling) of high-enrgy d.o.f. from low-ene…

Weak Quantum Gravity EFT fits nicely in this framework and behaves excellently in this regime of low energies (like a good quantum theory)



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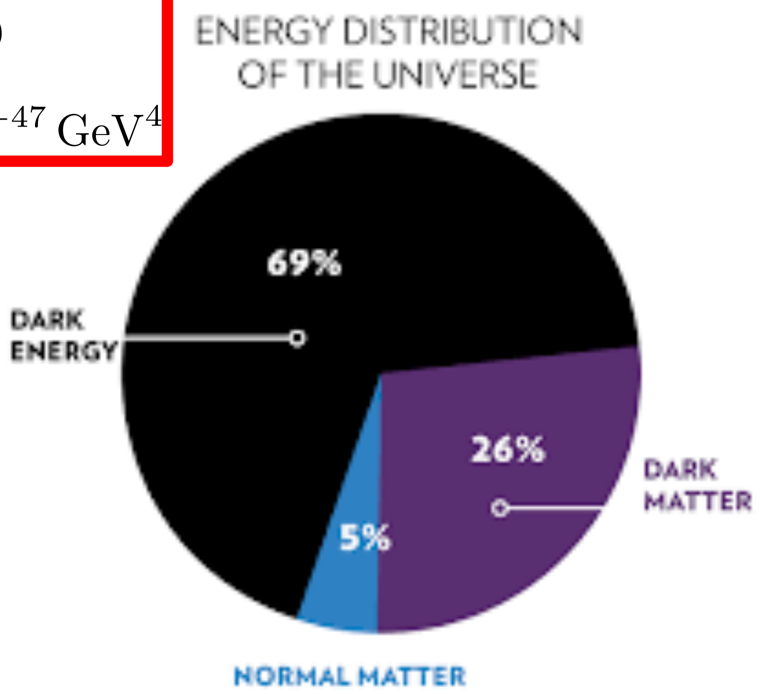
Quantum corrections in Derivative (energy) expansion for both pure gravity and matter

Set $\Lambda = 0$ today (**current cosmological measurements indicate** $\Lambda \approx 2.5 \times 10^{-47}$ GeV ^{4})

Not relevant in local tests of Gravity, but important for cosmological searches

$$\boxed{\begin{array}{l}\Omega_\Lambda = \dfrac{\rho_\Lambda}{\rho_c} \simeq 0.69 \\ \rho_c \simeq 3.68 \times 10^{-47}\,\mathrm{GeV}^4\end{array}}$$



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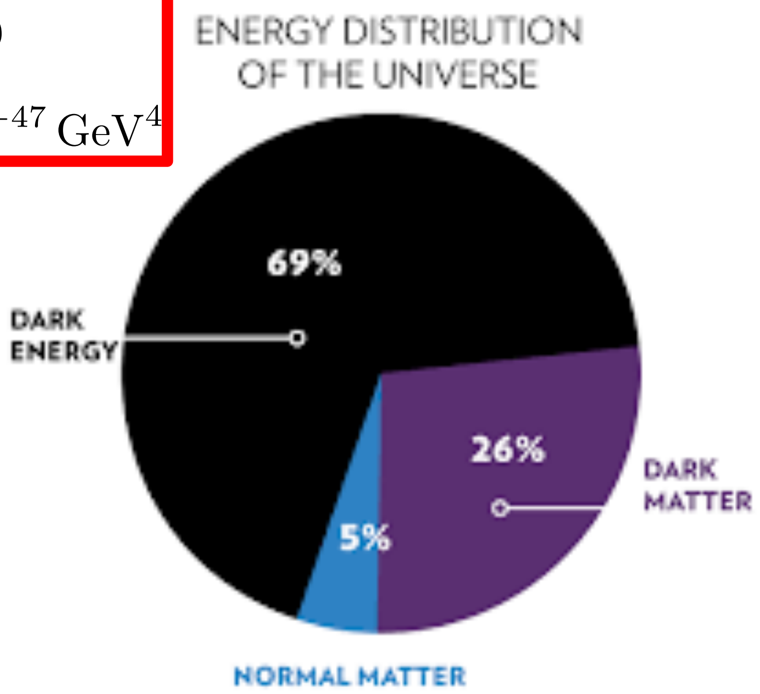

$$\mathcal{L}_g = \sqrt{g}\left\{\frac{2}{\kappa^2}R + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + \mathcal{O}(R^3)\right\}$$

$$\mathcal{L}_m = \sqrt{g}\left\{\frac{1}{2}(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2) + d_1 R^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + R(d_2 g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + d_3 m^2\phi^2) + \ldots\right\}$$

coefficients c_i : $[\, c_i \,] = 0$ → **matrix elements** will undergo the **expansion: $1 + G\, q^2 c_i$** $\approx 1 + \frac{q^2}{\Lambda^2_{\rm grav}}$ $\quad \Lambda_{\rm grav} = M_{\rm Pl}$

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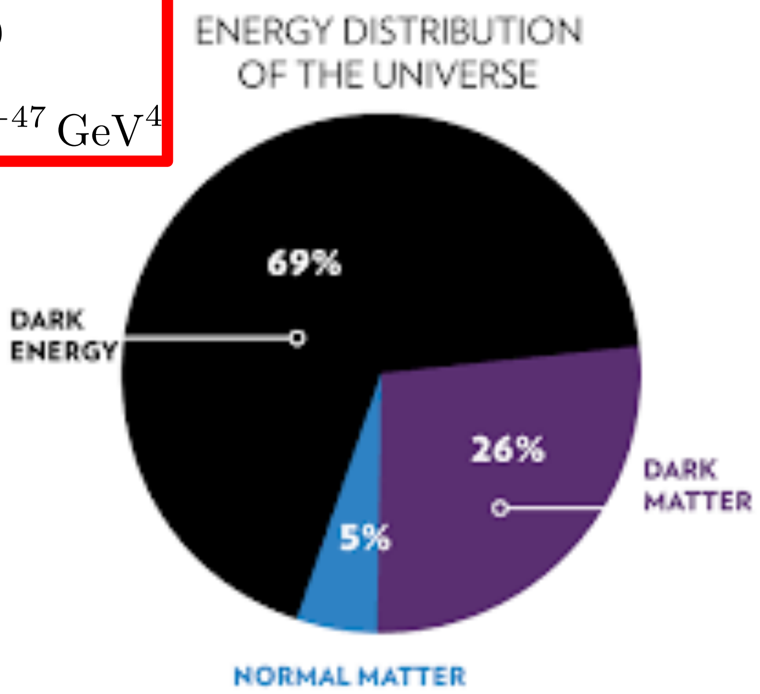

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$$\begin{aligned}\mathcal{L}_m \;=\; & \sqrt{g}\left\{\frac{1}{2}(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2)\right. \\ & \left. + d_1 R^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + R(d_2 g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + d_3 m^2\phi^2) + \ldots\right\}\end{aligned}$$

coefficients c_i : $[\,c_i\,] = 0$ → **matrix elements** will undergo the **expansion: $1 + G\, q^2 c_i$** $\approx 1 + \frac{q^2}{\Lambda^2_{\mathrm{grav}}}$ $\Lambda_{\mathrm{grav}} = M_{\mathrm{Pl}}$

coefficients d_i : $[\,d_i\,] = -2$ → but for composite matter bodies $O(d_i) = \langle r^2 \rangle$ (r = physical extent of the matter body)

d_i : play role analogous to charge radius in QED:
energy-momentum charge-radius quantum corrections of $O(\alpha/m_e^2)$

G. 't Hooft & M. Veltman, Ann. Inst. Henri Poincaré A20 (1974) , 69

J.F. Donoghue, PRD50 (1994), 3874, 9512024 [gr-qc]

Use of background equations of motion, after inclusion of matter

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} = 16\pi G \frac{\delta S_{\mathrm{eff,matter}}}{\delta \bar{g}^{\mu\nu}}\Big|_{h_{\alpha\beta}=0} .$$

yields the following form for weak-quantum gravity effective action (to quadratic order in graviton fluctuations $h_{\mu\nu}$, including FP GF terms) :

$$\boxed{S_{\mathrm{eff}} = \int d_x^4 \sqrt{\bar{g}} \Big\{ \bar{\mathcal{L}}(\bar{g}) - \frac{1}{2} h_{\alpha\beta} D^{\alpha\beta\gamma\delta} h_{\gamma\delta} + \cdots \Big\} + \eta^{*\mu} \Big\{ d_\lambda d^\lambda \; \bar{g}_{\mu\nu} - \bar{R}_{\mu\nu} \Big\} \eta^\nu + \mathcal{O}(h^3) \Big\}}$$

$$D^{\alpha\beta\gamma\delta} = I^{\alpha\beta,\mu\nu} d_\lambda d^\lambda I_{\mu\nu,}{}^{\gamma\delta} - \frac{1}{2}\bar{g}^{\alpha\beta} d_\lambda d^\lambda \bar{g}^{\gamma\delta} + \bar{g}^{\alpha\beta} d^\gamma d^\delta + d^\alpha d^\beta \bar{g}^{\gamma\delta} - 2 I^{\alpha\beta,\mu\nu} d_\sigma d_\lambda I_\mu{}^{\sigma,\gamma\delta}$$
$$+ \bar{R}\Big(I^{\alpha\beta,\gamma\delta} - \frac{1}{2} g^{\alpha\beta} g^{\gamma\delta} \Big) + \Big(\bar{g}^{\alpha\beta} \bar{R}^{\gamma\delta} + \bar{R}^{\alpha\beta} \bar{g}^{\gamma\delta} \Big) - 4 I^{\alpha\beta,\lambda\mu} \bar{R}_{\mu\nu} I_\lambda{}^{\nu,\gamma\delta}$$

$$I^{\alpha\beta,\gamma\delta} = \frac{1}{2}\left(\bar{g}^{\alpha\gamma}\bar{g}^{\beta\delta} + \bar{g}^{\alpha\delta}\bar{g}^{\beta\gamma}\right)$$ d_μ being a covariant derivative w.r.t. background

The quantum one-loop path Integral in a spacetime background $g_{\mu\nu}$

$$\begin{aligned} Z[\bar{g}] &= \int [dh_{\mu\nu}]\, exp\left\{ i \int d^4x \sqrt{\bar{g}} \left\{ \frac{2}{\kappa^2} \bar{R} + h_{\mu\nu} D^{\mu\nu\alpha\beta} h_{\alpha\beta} \right\} \right. \\ &= det D^{\mu\nu\alpha\beta} \\ &= expTrln(D^{\mu\nu\alpha\beta}) \end{aligned}$$

Dimensional Regularization to handle UV divergencies $\epsilon = 4 - d$

G. 't Hooft & M. Veltman, Ann. Inst. Henri Poincaré A20 (1974) , 69

Effective Lagrangian (1-loop UV divergent) in pure gravity:

$$\mathcal{L}^{(div)}_{1loop} = \frac{1}{8\pi^2\epsilon} \left\{ \frac{1}{120} \bar{R}^2 + \frac{7}{20} \bar{R}_{\mu\nu} \bar{R}^{\mu\nu} \right\}$$

matter fields also contribute such terms

J.F. Donoghue, PRD50 (1994), 3874, 9512024 [gr-qc]

So renormalized (r) c_i, $i=1,2$ coefficients $c_1^{(r)} = c_1 + \frac{1}{960\pi^2\epsilon}$, $c_2^{(r)} = c_2 + \frac{7}{160\pi^2\epsilon}$

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NB: Pure gravity @ 2-loops : $\quad \mathcal{L}^{(div)}_{2loop} = \frac{209\kappa^2}{2880(16\pi^2)}\frac{1}{\epsilon}\bar{R}^{\alpha\beta}{}_{\gamma\delta}\bar{R}^{\gamma\delta}{}_{\eta\sigma}\bar{R}^{\eta\sigma}{}_{\alpha\beta}$

M. Goroff, A. Sagnotti Nucl. Phys. B266 (1986), 799

The quantum **one-loop** path Integral in a spacetime background $g_{\mu\nu}$

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&= det D^{\mu\nu\alpha\beta} \\
&= exp Tr ln(D^{\mu\nu\alpha\beta})
\end{aligned}
$$

Dimensional Regularization to han...

Effective Lagrangi...

$$\mathcal{L}^{(div)}_{1loop} = \frac{1}{8\pi^2\epsilon} \{ \ldots$$

So renormalized (r) ...

$$\ldots \frac{\ldots}{960\pi^2\epsilon} \;, \quad c_2^{(r)} = c_2 + \frac{7}{160\pi^2\epsilon}$$

Higher loops carry more powers of Curvature as a consequence of general covariance and structure of **energy expansion** in a **massless** theory

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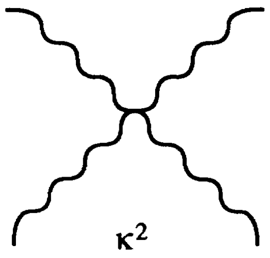
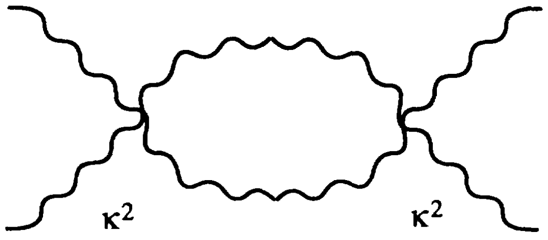
J.F. Donoghue, PRD50 (1994), 3874, 9512024 [gr-qc]

M. Goroff, A. Sagnotti Nucl. Phys. B266 (1986), 799

Why? simple example: four-graviton vertex @ tree-level (a) and 1-loop level (b)



(a)



(b)

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa h_{\mu\nu} \ ,$$
$$g^{\mu\nu} = \bar{g}^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu}_{\lambda} h^{\lambda\nu} + \cdots$$

Each graviton brings a factor $\kappa = M_{Pl}^{-1}$ → **(a) : contribution of Einstein-Hilbert term in the action:** $\boxed{M_{\rm Ein} \sim \kappa^2 p^2}$

(p a representative of external momenta)

lagrangian at order E^4 behaves as: $\boxed{M_{\rm HO} \sim c_1 \kappa^4 p^4}$

Loop correction (b):

$$\boxed{\begin{aligned} M_{\rm loop} &\sim \int d^4l\, \kappa^2 l^2 \frac{1}{l^2} \frac{1}{(l-p)^2} \kappa^2 l^2 \\ &\sim \kappa^4 I(p) \sim \kappa^4 p^4 \ . \end{aligned}}$$

quadratically UV divergent internal loop

Needs regularization/renormalization dimensional regularizaton preserves general covariance → only logarithms not powers of momenta → final momentum power of a graph → counting powers of κ (no mass scale in pure gravity)

Strategy to deal with 1-loop weak-gravity effective field theory coupled to matter

J.F. Donoghue, PRD50 (1994), 3874,

(1) Define the quantum degrees of freedom using the lowest order $[O(p^2)]$ effective Lagrangian,

(2) Calculate the one-loop corrections.

(3) Combine the effects of the order p^2 and p^4 Lagrangians (given earlier in this section) at the tree level with the one-loop corrections. The divergences (and some accompanying finite parts) of the loop diagrams may be absorbed into renormalized coefficients of the Lagrangian (m, c_i, d_i), using some renormalization scheme which does not violate general covariance.

(4) Measure the unknown coefficients by comparison with some experimental measurement.

(5) Having determined the parameters of the theory, one can make predictions for other experimental observables, valid to $O(p^4)$ in the energy expansion.

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Difficult part due to weakness of gravity

APPLICATION:

Weak-Quantum-Gravity Corrections
to
(low-energy, Newtonian) Gravitational Potentials
between two masses m_1 , m_2

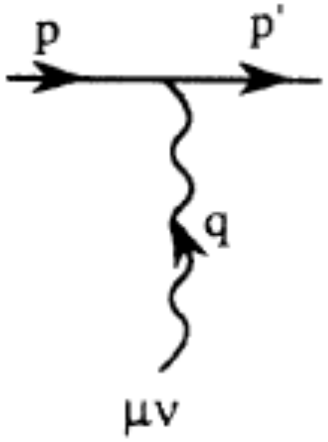
Feynman Rules for flat background $g_{\mu\nu} = \eta_{\mu\nu}$ (of interest to us)

Graviton propagator in harmonic gauge:

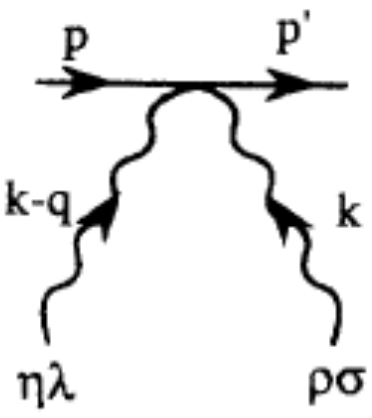
$$iD_{\mu\nu\alpha\beta}(q) = \frac{i}{q^2 - i\epsilon} P_{\mu\nu,\alpha\beta}$$

$$P_{\mu\nu,\alpha\beta} = \frac{1}{2}\left[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta} - \eta_{\mu\nu}\eta_{\alpha\beta}\right]$$

Matter-graviton vertices :



(a)



(b)

Fig. (a) : $$\tau_{\mu\nu}(p,p') = \frac{-i\kappa}{2}\left(p_\mu p'_\nu + p'_\mu p_\mu - g_{\mu\nu}[p\cdot p' - m^2]\right)$$

Fig. (b) : $$V_{\eta\lambda,\rho\sigma} = \frac{i\kappa^2}{2}\Big\{I_{\eta\lambda,\alpha\delta}I^{\delta}{}_{\beta,\rho\sigma}\left(p^\alpha p'^\beta + p^\alpha p'^\beta\right) - \frac{1}{2}\left(\eta_{\eta\lambda}I_{\rho\sigma,\alpha\beta} + \eta_{\rho\sigma}I_{\eta\lambda,\alpha\beta}\right)p'^\alpha p^\beta$$
$$-\frac{1}{2}\left(I_{\eta\lambda,\rho\sigma} - \frac{1}{2}\eta_{\eta\lambda}\eta_{\rho\sigma}\right)[p\cdot p' - m^2]\Big\} ,$$

$$I_{\alpha\beta,\gamma\delta} \equiv \frac{1}{2}\left(\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma}\right) .$$

Graviton three-vertex

$$\mathcal{L}_g^{(1)} = \frac{h_{\mu\nu}}{\kappa}\left[\bar{g}^{\mu\nu}\bar{R} - 2\bar{R}^{\mu\nu}\right] ,$$

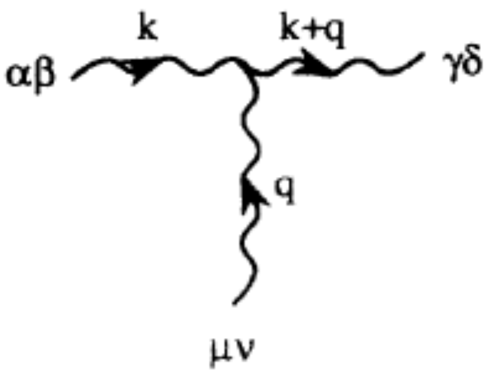
$$\mathcal{L}_g^{(2)} = \frac{1}{2}h_{\mu\nu;\alpha}h^{\mu\nu;\alpha} - \frac{1}{2}h_{;\alpha}h^{;\alpha} + h_{;\alpha}h^{\alpha\beta}{}_{;\beta} - h_{\mu\beta;\alpha}h^{\mu\alpha;\beta} + \bar{R}\left(\frac{1}{4}h^2 - \frac{1}{2}h_{\mu\nu}h^{\mu\nu}\right) + \left(2h^\lambda_\mu h_{\nu\lambda} - hh_{\mu\nu}\right)\bar{R}^{\mu\nu} .$$

$$\mathcal{L}_{GF} = \sqrt{-\bar{g}}\left\{\left(h_{\mu\nu}{}^{;\nu} - \frac{1}{2}h_{;\mu}\right)\left(h^{\mu\lambda}{}_{;\lambda} - h^{;\mu}\right)\right\}$$

expand background metric

$$\bar{g}(x) = \eta_{\mu\nu} + \kappa H^{\text{ext}}_{\mu\nu}(x)$$

Pick up one external and two internal (quantum) fields

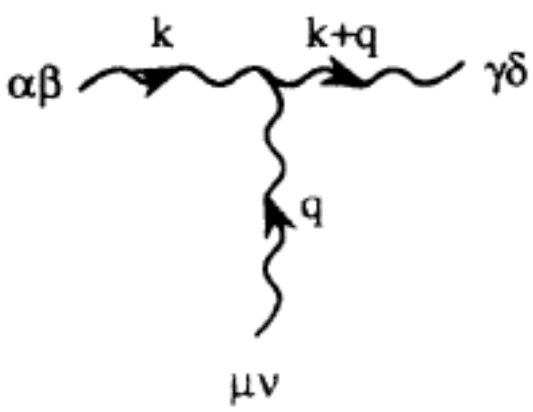


(c)

Graviton three-vertex

Feynman Rule

$$
\begin{aligned}
\tau^{\mu\nu}_{\alpha\beta,\gamma\delta} = \frac{i\kappa}{2}\Bigg(& P_{\alpha\beta,\gamma\delta}\left[k^\mu k^\nu + (k-q)^\mu (k-q)^\nu + q^\mu q^\nu - \frac{3}{2}\eta^{\mu\nu} q^2\right] \\
& + 2q_\lambda q_\sigma \left[I^{\lambda\sigma,}{}_{\alpha\beta} I^{\mu\nu,}{}_{\gamma\delta} + I^{\lambda\sigma,}{}_{\gamma\delta} I^{\mu\nu,}{}_{\alpha\beta} - I^{\lambda\mu,}{}_{\alpha\beta} I^{\sigma\nu,}{}_{\gamma\delta} - I^{\sigma\nu,}{}_{\alpha\beta} I^{\lambda\mu,}{}_{\gamma\delta}\right] \\
& + \Big[q_\lambda q^\mu \left(\eta_{\alpha\beta} I^{\lambda\nu,}{}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\nu,}{}_{\alpha\beta}\right) + q_\lambda q^\nu (\eta_{\alpha\beta} I^{\lambda\mu,}{}_{\gamma\delta} + \eta_{\gamma\delta} I^{\lambda\mu,}{}_{\alpha\beta}) \\
& - q^2 (\eta_{\alpha\beta} I^{\mu\nu,}{}_{\gamma\delta} + \eta_{\gamma\delta} I^{\mu\nu,}{}_{\alpha\beta}) - \eta^{\mu\nu} q^\lambda q^\sigma (\eta_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} + \eta_{\gamma\delta} I_{\alpha\beta,\lambda\sigma})\Big] \\
& + \Big[2q^\lambda (I^{\sigma\nu,}{}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} (k-q)^\mu + I^{\sigma\mu,}{}_{\alpha\beta} I_{\gamma\delta,\lambda\sigma} (k-q)^\nu - I^{\sigma\nu,}{}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\mu - I^{\sigma\mu,}{}_{\gamma\delta} I_{\alpha\beta,\lambda\sigma} k^\nu) \\
& + q^2 (I^{\sigma\mu,}{}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\nu + I_{\alpha\beta,\sigma}{}^\nu I^{\sigma\mu,}{}_{\alpha\delta}) + \eta^{\mu\nu} q^\lambda q_\sigma (I_{\alpha\beta,\lambda\rho} I^{\rho\sigma,}{}_{\gamma\delta} + I_{\gamma\delta,\lambda\rho} I^{\rho\sigma,}{}_{\alpha\beta})\Big] \\
& + \Big\{(k^2 + (k-q)^2)\left(I^{\sigma\mu,}{}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\nu + I^{\sigma\nu,}{}_{\alpha\beta} I_{\gamma\delta,\sigma}{}^\mu - \frac{1}{2}\eta^{\mu\nu} P_{\alpha\beta,\gamma\delta}\right) \\
& - (k^2 \eta_{\gamma\delta} I^{\mu\nu,}{}_{\alpha\beta} + (k-q)^2 \eta_{\alpha\beta} I^{\mu\nu,}{}_{\gamma\delta})\Big\}\Bigg)
\end{aligned}
$$



(c)

$$P_{\mu\nu,\alpha\beta} = \frac{1}{2}\left[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta} - \eta_{\mu\nu}\eta_{\alpha\beta}\right]$$

$$I_{\alpha\beta,\gamma\delta} \equiv \frac{1}{2}\left(\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma}\right)$$

Applications: Tree-level Newtonian Potential of two masses m_1 , m_2

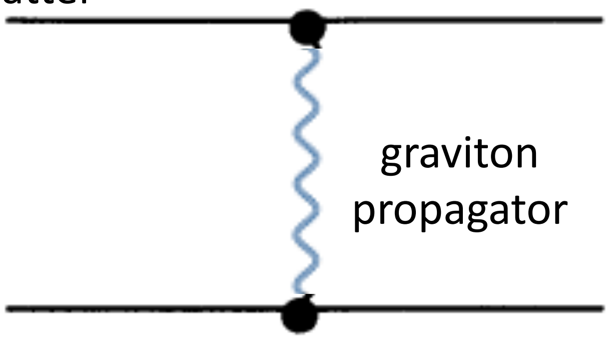


Fig 1

Tree level graviton propagator

$$iD_{\mu\nu\alpha\beta}(q) = \frac{i}{q^2 - i\epsilon} P_{\mu\nu,\alpha\beta}$$

$$P_{\mu\nu,\alpha\beta} = \frac{1}{2}\left[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta} - \eta_{\mu\nu}\eta_{\alpha\beta}\right]$$

Matter stress tensor $T_{\mu\nu}$ matrix element :

$$V_{0\mu\nu}(q) \equiv \langle p' \mid T_{\mu\nu} \mid p\rangle$$
$$= p_\mu p'_\nu + p'_\mu p_\nu + \frac{1}{2} q^2 \eta_{\mu\nu}$$

used State normalization condition: $\langle p' \mid p\rangle = 2E(2\pi)^3\delta^3(p - p')$

Applications: Tree-level Newtonian Potential of two masses m_1 , m_2

Graviton exchange @ tree level $M_{12} = \frac{\kappa^2}{4} V^{(1)}_{0\mu\nu}(q) D^{\mu\nu,\alpha\beta}(q) V^{(2)}_{0\alpha\beta}(-q).$

The static limit corresponds to $q_\mu = (0, \mathbf{q})_\mu$, $\frac{1}{2m_1} V^{(1)}_{\mu\nu}(q) = m_1 \delta_{\mu 0} \delta_{\nu 0}$

Fourier transform of $\frac{1}{2m_1 2m_2} M_{12} \sim -\frac{\kappa^2}{8} \frac{m_1 m_2}{\mathbf{q}^2}$ → Newtonian potential:

$$\boxed{V(r) = -\int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{r}} \frac{\kappa^2}{8} \frac{m_1 m_2}{\mathbf{q}^2} = -G\frac{m_1 m_2}{r}.}$$

Standard classical result

Applications: Quantum Graviton 1-loop Corrections to the Newtonian Potential of two masses m_1, m_2

G. 't Hooft & M. Veltman,
Ann. Inst. Henri Poincaré
A20 (1974) , 69

J.F. Donoghue,
PRD50 (1994), 3874,
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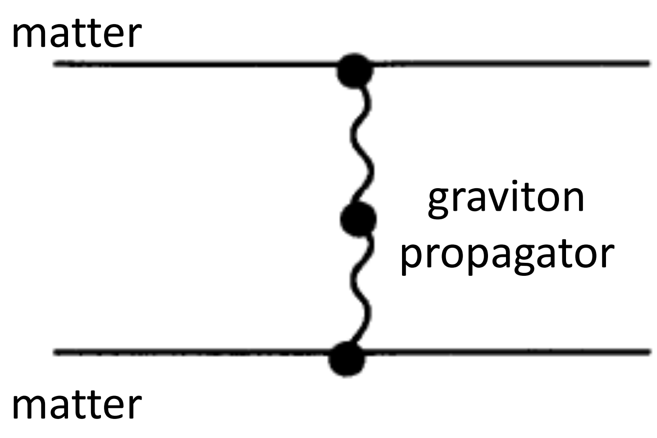


Fig 2

Fig 3

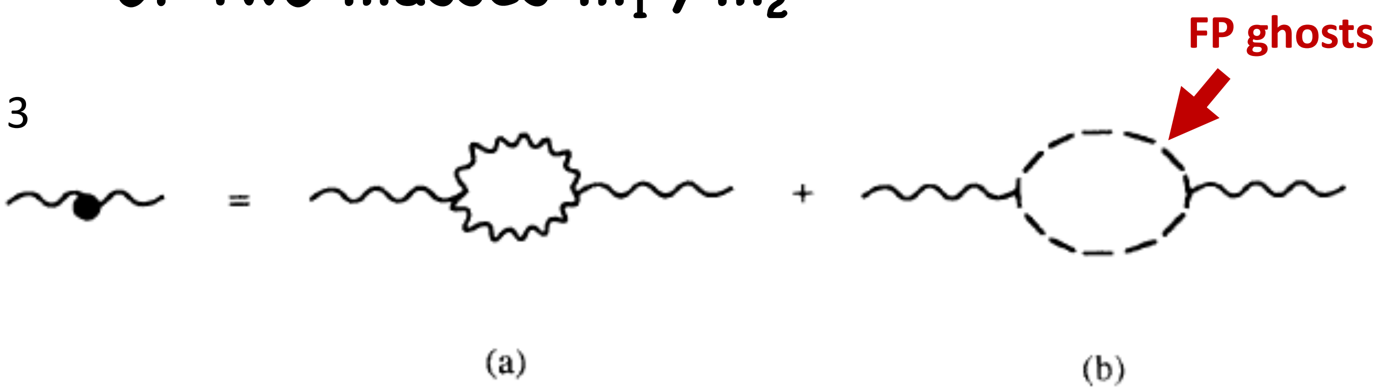
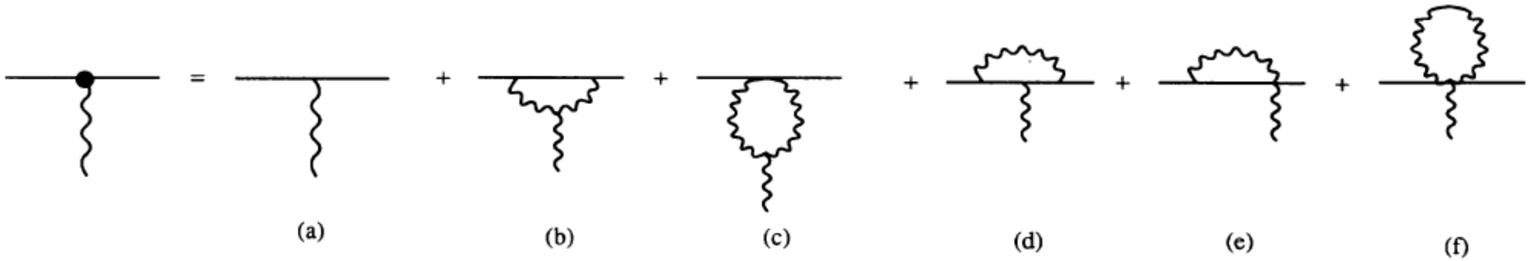


Fig 4



Form of Corrections :

$$\boxed{V(r) = -\frac{Gm_1m_2}{r}\left[1 + a\frac{G(m_1+m_2)}{rc^2} + b\frac{G\hbar}{r^2c^3} + \cdots\right]}$$

long-range propagation of massless particles

Applications: Quantum Graviton 1-loop Corrections to the Newtonian Potential of two masses m_1 , m_2

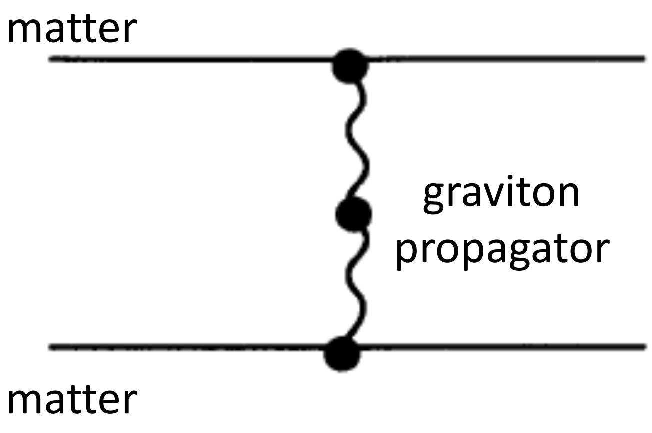


Fig 2

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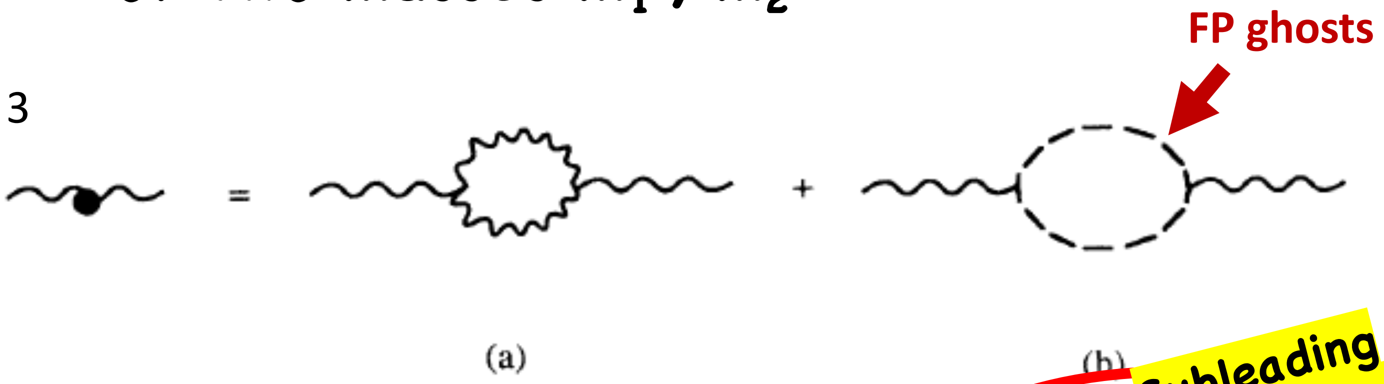
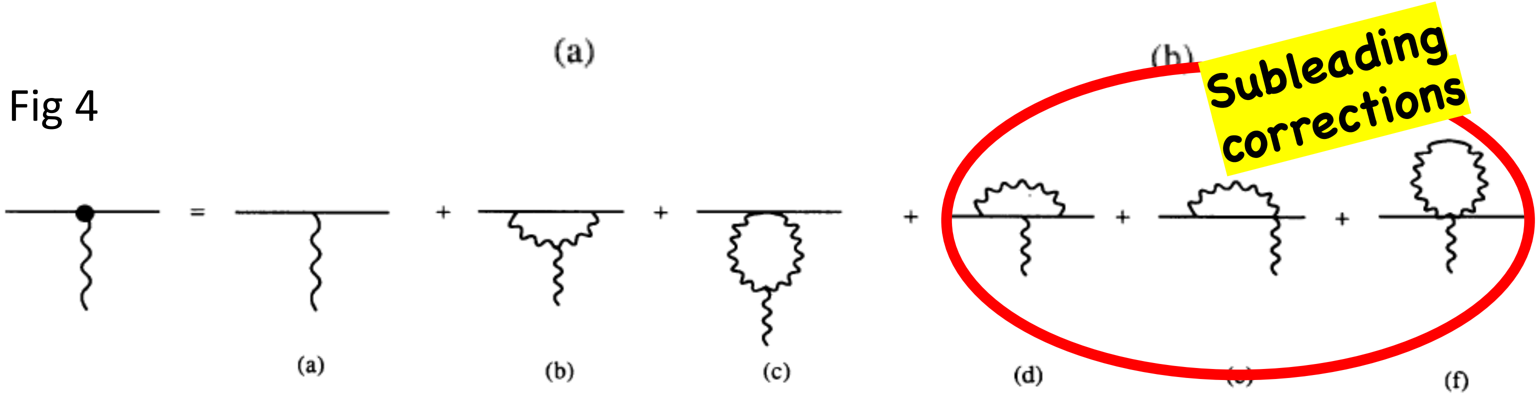


Fig 4



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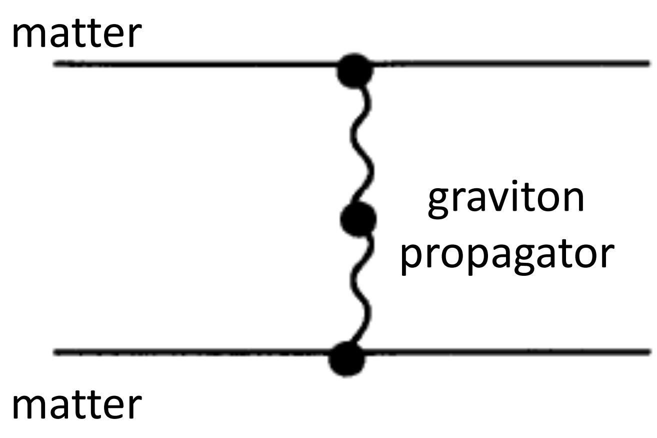

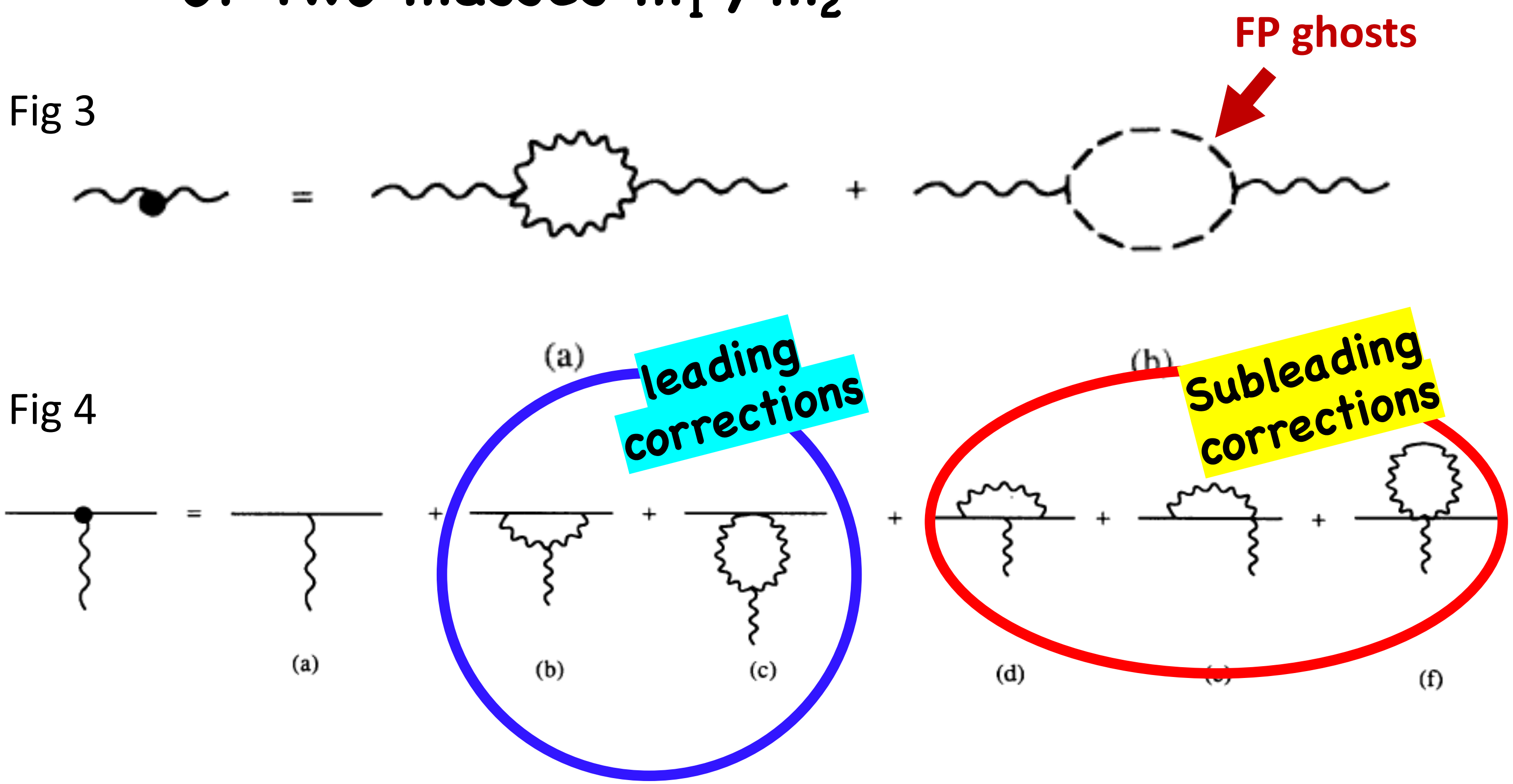

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long-range propagation of massless particles

$$M^{\mu\nu} \text{ (4b)} = \int \frac{d^4k}{(2\pi)^4} i\tau_{\eta\lambda}(p, p'-k) \frac{i}{(k-p')^2 - m^2 + i\epsilon} i\tau_{\rho\sigma}(p'-k, p') iD^{\eta\lambda,\alpha\beta}(k-q) i\tau^{\mu\nu}_{\alpha\beta,\gamma\delta} iD^{\gamma\delta,\rho\sigma}(k),$$

$$M^{\mu\nu} \text{ (4c)} = \int \frac{d^4k}{(2\pi)^4} iV_{\eta\lambda,\rho\sigma} iD^{\eta\lambda,\alpha\beta}(k-q) i\tau^{\mu\nu}_{\alpha\beta,\gamma\delta} iD^{\gamma\delta,\rho\sigma}(k) \, .$$

Lorenz structure of the vertex

$$\boxed{\begin{aligned} V_{\mu\nu} &\equiv \langle p' \mid T_{\mu\nu} \mid p \rangle \\ &= F_1(q^2)\left[p_\mu p'_\nu + p'_\mu p_\nu + q^2 \frac{\eta_{\mu\nu}}{2}\right] \\ &\quad + F_2(q^2)\left[q_\mu q_\nu - g_{\mu\nu} q^2\right] \end{aligned}}$$

Normalization F_1 (0)

$$F_1(q^2) = 1 + d_1 q^2 + \kappa^2 q^2 \left(\ell_1 + \ell_2 \ln \frac{(-q^2)}{\mu^2} + \ell_3 \sqrt{\frac{m^2}{-q^2}}\right) + \cdots \, ,$$

$$F_2(q^2) = -4(d_2 + d_3) m^2 + \kappa^2 m^2 \left(\ell_4 + \ell_5 \ln \frac{(-q^2)}{\mu^2} + \ell_6 \sqrt{\frac{m^2}{-q^2}}\right) + \cdots \, ,$$

μ : renormalisation-group transmutation mass

constants ℓ_1 and ℓ_4 UV divergent
come from high-energy end of loop integrals
ℓ_2, ℓ_3, ℓ_5, and ℓ_6 must be finite

$\ln(-q^2)$ and $\sqrt{\frac{m^2}{-q^2}}$ → imaginary

for q^2 >0 (timelike) → Unitarity

d_i associated with unknown
UV-complete high-energy theory

(The ellipses ...
denote higher powers of q^2)

Conjectures because we do not know the UV completion of weak gravity:

Combine the constants ℓ_1 and ℓ_4 with the constants d_i to renormalized (r) quantities:

$$\boxed{\begin{aligned} d_1^{(r)}(\mu^2) &= d_1 + \kappa^2 \ell_1 \;, \\ d_2^{(r)}(\mu^2) + d_3^{(r)}(\mu^2) &= d_1 + d_3 - \kappa^2 \frac{\ell_4}{4} \;. \end{aligned}}$$

μ : renormalisation-group transmutation mass,
Physical quantities independent of μ

c.f.

$$\mathcal{L}_g = \sqrt{g}\left\{\frac{2}{\kappa^2}R + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + \mathcal{O}(R^3)\right\}$$

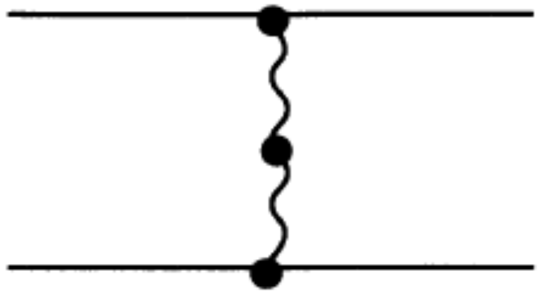
$$\begin{aligned} \mathcal{L}_m &= \sqrt{g}\left\{\frac{1}{2}(g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2)\right. \\ &\quad \left. + d_1 R^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + R(d_2 g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + d_3 m^2\phi^2) + \ldots\right\} \end{aligned}$$

...after several algebraically involved calculations, we obtain for the gravitational potential between two masses:

J.F. Donoghue, PRD50 (1994), 3874, 9512024 [gr-qc]

$$-\frac{\kappa^2}{4}\frac{1}{2m_1}V^{(1)}_{\mu\nu}(q)\left[iD^{\mu\nu,\alpha\beta}(q)+iD^{\mu\nu,\rho\sigma}i\Pi_{\rho\sigma,\eta\lambda}iD^{\eta\lambda,\alpha\beta}\right]V_{\alpha\beta}(q)\frac{1}{2m_2}$$

$$\approx 4\pi G m_1 m_2\left[\frac{i}{\mathbf{q}^2}-\frac{i\kappa^2}{32\pi^2}\left[-\frac{127}{60}\ln(\mathbf{q}^2)+\frac{\pi^2(m_1+m_2)}{2\sqrt{\mathbf{q}^2}}\right]+\text{const}\right]$$



1-loop weak quantum gravity corrections to Newtonian potential between masses m_1, m_2

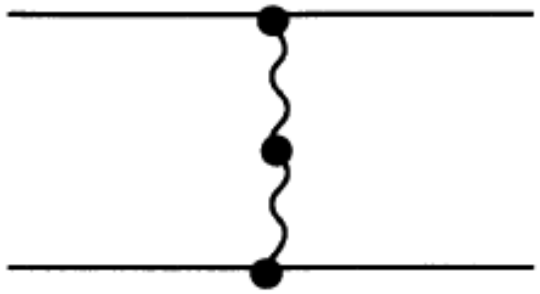
$$\boxed{V(r)=-\frac{Gm_1m_2}{r}\left[1-\frac{G(m_1+m_2)}{rc^2}-\frac{127}{30\pi^2}\frac{G\hbar}{r^2c^3}\right]}$$

...after several algebraically involved calculations, we obtain for the gravitational potential between two masses:

J.F. Donoghue, PRD50 (1994), 3874, 9512024 [gr-qc]

$$-\frac{\kappa^2}{4}\frac{1}{2m_1}V^{(1)}_{\mu\nu}(q)\left[iD^{\mu\nu,\alpha\beta}(q)+iD^{\mu\nu,\rho\sigma}i\Pi_{\rho\sigma,\eta\lambda}iD^{\eta\lambda,\alpha\beta}\right]V_{\alpha\beta}(q)\frac{1}{2m_2}$$

$$\approx 4\pi G m_1 m_2\left[\frac{i}{\mathbf{q}^2}-\frac{i\kappa^2}{32\pi^2}\left[-\frac{127}{60}\ln(\mathbf{q}^2)+\frac{\pi^2(m_1+m_2)}{2\sqrt{\mathbf{q}^2}}\right]+\mathrm{const}\right]$$



1-loop weak quantum gravity corrections to Newtonian potential between masses m_1, m_2

$$V(r)=-\frac{Gm_1m_2}{r}\left[1-\frac{G(m_1+m_2)}{rc^2}-\frac{127}{30\pi^2}\frac{G\hbar}{r^2c^3}\right]$$

Integrals needed for the calculation of the 1-loop quantum corrections to the Newtonian potential between two masses m_1 , m_2

J.F. Donoghue, PRD50 (1994), 3874, 9512024 [gr-qc]

$$q^2 = q_0^2 - \mathbf{q}^2 \qquad\qquad L = \ln(-q^2), S = \pi^2 m/\sqrt{-q^2}$$

external momentum p' is on shell as is $p = p' - q$

$$J = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k-q)^2} = \frac{i}{32\pi^2}[-2L] + \cdots,$$

$$J_\mu = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu}{k^2(k-q^2)} = \frac{i}{32\pi^2} q_\mu [-L] + \cdots,$$

$$J_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu}{k^2(k-q^2)} = \frac{i}{32\pi^2}\left[q_\mu q_\nu \left\{-\frac{2}{3}L\right\} - g_{\mu\nu}\, q^2 \left\{-\frac{1}{6}L\right\}\right] + \cdots$$

$$I = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{(k-q)^2} \frac{1}{[(k-p')^2 - m^2]} = \frac{i}{32\pi^2 m^2}[-L - S] \, ,$$

$$I_\mu = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu}{k^2(k-q)^2[(k-p')^2 - m^2]} = \frac{i}{32\pi^2 m^2}\left[p'_\mu\left\{\left(1 + \frac{q^2}{2m^2}\right)L + \frac{1}{4}\frac{q^2}{m^2}S\right\} + q_\mu\left\{-L - \frac{1}{2}S\right\}\right] \, ,$$

$$I_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu}{k^2(k-q)^2\,[(k-p')^2 + m^2]} = \frac{i}{32\pi^2 m^2}\left[p'_\mu p'_\nu\left\{-\frac{q^2}{2m^2}L\right.\right.$$

$$\left.\left. -\frac{q^2}{8m^2}S\right\} + (p'_\mu q_\nu + p'_\nu q_\mu)\left\{\frac{1}{2}\left(1 + \frac{q^2}{m^2}\right)L + \frac{3}{16}\frac{q^2}{m^2}S\right\} + q_\mu q_\nu\left\{-L - \frac{3}{8}S\right\} + q^2 g_{\mu\nu}\left\{\frac{1}{4}L + \frac{1}{8}S\right\}\right] + \cdots$$

$$I_{\mu\nu\alpha} = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu k_\alpha}{k^2(k-q)^2\,[(k-p')^2 + m^2]}$$

$$= \frac{i}{32\pi^2 m^2}\left\{\left[p'_\mu p'_\nu p'_\alpha\left\{\frac{1}{6}\frac{q^2}{m^2}L\right]\right\} + (p'_\mu p'_\nu q_\alpha + p'_\mu q_\nu p'_\alpha + q_\mu p'_\nu p'_\alpha)\left\{-\frac{1}{3}\frac{q^2}{m^2}L - \frac{q^2}{16m^2}S\right\}\right.$$

$$+(q_\mu q_\nu p'_\alpha + p'_\mu q_\nu q'_\alpha + q_\mu p'_\nu q_\alpha)\left[\frac{1}{3}L\right] + q_\mu q_\nu q_\alpha\left[-L - \frac{5}{16}S\right]$$

$$\left. +\left(g_{\mu\nu}p'_\alpha + g_{\mu\alpha}p'_\nu + g_{\nu\alpha}p'_\mu\right)\left[-\frac{q^2}{12}L\right] + \left(g_{\mu\nu}q_\alpha + g_{\mu\alpha}q_\nu + g_{\nu\alpha}q_\mu\right)\left[\frac{q^2 L}{6} + \frac{q^2 S}{16}\right]\right\} + \cdots \, .$$

Some steps of the computation

Without vacuum polarization

$$\frac{\kappa^2}{4}V^{(1)}_{\mu\nu}(q)D^{\mu\nu,\alpha\beta}(q)V^{(2)}_{\alpha\beta}(-q) = \frac{\kappa^2}{2q^2}\Big[F_1^{(1)}(q^2)F_1^{(2)}(q^2)\Big\{p_1\cdot p_2 p_1'\cdot p_2' + p_1\cdot p_2' p_2\cdot p_1' - m_1^2 m_2^2\Big\}$$

$$+\frac{q^2}{2}\Big\{F_1^{(1)}(q^2)F_2^{(2)}(q^2)m_1^2 + F_1^{(2)}(q^2)F_2^{(1)}(q^2)m_2^2\Big\}\Big]$$

approximate result in static limit

$$\approx \frac{\kappa^2 m_1^2 m_2^2}{2}\Big\{\frac{1}{q^2} + 2\left(d_1 - 2d_2 - 2d_3\right)$$

$$+\kappa^2\Big[(2\ell_1 - \ell_4) + (2\ell_2 - \ell_5)\ln\left(\frac{-q^2}{\mu^2}\right) + (2\ell_3 - \ell_6)\sqrt{\frac{m^2}{-q^2}}\Big]\Big\},$$

To get to configuration space, one shall need :

$$\int \frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}} = \delta^3(x), \qquad \int \frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}\frac{1}{q} = \frac{1}{2\pi^2 r^2}$$

$$\int \frac{d^3q}{(2\pi)^3}e^{-i\mathbf{q}\cdot\mathbf{r}}\ln\mathbf{q}^2 = \frac{-1}{2\pi^2 r^3}$$

Including vacuum polarization

generic expected structure
(dimensional counting)

$$\pi(q^2) = \kappa^2 q^4 \left[c_1 + c_2 + \ell_7 + \ell_8 \ln(-q^2)\right]$$

graviton-propagator modifications

$$\frac{1}{q^2} + \frac{1}{q^2}\pi(q^2)\frac{1}{q^2} + \cdots = \left\{\frac{1}{q^2} + \kappa^2[c_1 + c_2 + \ell_7 + \ell_8 \ln(-q^2)]\right\}.$$

(**cf.** $\mathcal{L}_{g4} = c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu}$, $c_{1,2}$ unknown coefficients from higher-order Lagrangian)

ℓ_7, ℓ_8 are constants **caclulable in vacuum polarization graph**

ℓ_7 is **UV divergent** but the combination $(c_1 + c_2 + \ell_7)$ leads to a **renormalizable parameter** (in principle measurable)

Using the fact that any factor of k^2 or $(k - q)^2$ in numerator removes non-analytic behaviour non external momenta p, e.g.

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k-q^2)} \frac{k^2}{(k-p')^2 - m^2} = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k-q)^2} \frac{1}{(k-p')^2 - m^2}$$

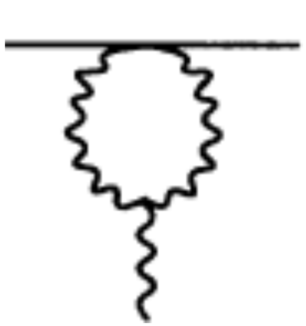
$$= \int \frac{d^4k'}{(2\pi)^4} \frac{1}{k'^2} \frac{1}{(k'-p)^2 - m^2} = I(p^2) .$$

The result is a function of m^2 only . On redefining the variable $k - q = k'$ we eliminate any q^2 dependence.

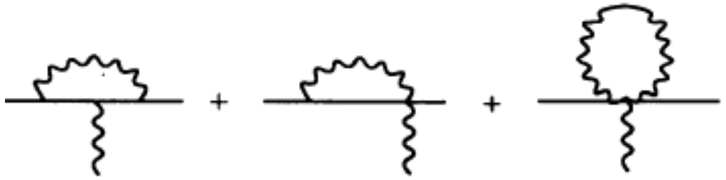
Also: $\boxed{q_\mu \tau^{\mu\nu}_{\alpha\beta,\gamma\delta} = 0}$

NB:

$$\Delta F_1(q^2) = \frac{\kappa^2 q^2}{32\pi^2}\left\{\left[\frac{1}{4} - 2 + 1 + 0\right]\ln(-q^2) + \left[\frac{1}{16} - 1 + 1 + 0\right]\frac{\pi^2 m}{\sqrt{-q^2}}\right\}$$
$$= \frac{\kappa^2 q^2}{32\pi^2}\left\{-\frac{3}{4}\ln(-q^2) + \frac{1}{16}\frac{\pi^2 m}{\sqrt{-q^2}}\right\},$$
$$\Delta F_2 = \frac{\kappa^2 m^2}{32\pi^2}\left\{[1 - 3 + 8 - 3]\ln(-q^2) + \left[\frac{7}{8} - 1 + 2 - 1\right]\frac{\pi^2 m}{\sqrt{-q^2}}\right\}$$
$$= \frac{\kappa^2 m^2}{32\pi^2}\left\{3\ln(-q^2) + \frac{7}{8}\frac{\pi^2 m^2}{\sqrt{-q^2}}\right\},$$



$$\Delta F_1 = \frac{\kappa^2 q^2}{32\pi^2}[0 + 2 + 0 - 2]\ln(-q^2)$$
$$= 0$$
$$\Delta F_2 = \frac{\kappa^2 m^2}{32\pi^2}\left[-\frac{25}{3} + 0 + 2 + 2\right]\ln(-q^2)$$
$$= \frac{\kappa^2 m^2}{32\pi^2}\left[-\frac{13}{3}\ln(-q^2)\right].$$



No non-analytic terms due to mass m of scalar field

Contains Infrared divergencies can be handled like in QED by adding soft radiative terms

Hence non-analytic contributions yield :

$$F_1(q^2) = 1 + \frac{\kappa^2}{32\pi^2}q^2\left[-\frac{3}{4}\ln(-q^2) + \frac{1}{16}\frac{\pi^2 m}{\sqrt{-q^2}}\right]$$
$$F_2(q^2) = \frac{\kappa^2 m^2}{32\pi^2}\left[-\frac{4}{3}\ln(-q^2) + \frac{7}{8}\frac{\pi^2 m}{\sqrt{-q^2}}\right].$$

Dimensional regularization → e.g. vacuum polarization involving massless internal particles

$$I(q^2) = \kappa^2 \mu^{4-d} \int \frac{d^d k}{(2\pi)^d} f(k,q) = \kappa^2 q^4 \left(\frac{\mu}{q}\right)^{4-d} \left[\frac{a}{d-4} + b\right]$$

(μ = transmutation mass scale , $\epsilon = 4 - d$)

$$\frac{1}{d-4}\left(\frac{\mu}{q}\right)^{4-d} = \frac{1}{d-4} e^{\frac{d-4}{2}\ln\left(\frac{q^2}{\mu^2}\right)}$$

$$= \frac{1}{d-4} + \frac{1}{2}\ln\left(\frac{q^2}{\mu^2}\right) + O(d-4)$$

G. 't Hooft & M. Veltman, Ann. Inst. Henri Poincaré A20 (1974) , 69

@ 1-loop (as mentioned above) :

$$\Delta\mathcal{L}_0^{(1)} = \frac{1}{8\pi^2}\frac{1}{\epsilon}\left\{\frac{1}{120}R^2 + \frac{7}{20}R_{\mu\nu}R^{\mu\nu}\right\}$$

This is Equivalent to log-part of the following graph:

J.F. Donoghue, PRD50 (1994), 3874, 9512024 [gr-qc]

$$\Pi_{\alpha\beta,\gamma\delta} = -\frac{\kappa^2}{32\pi^2}\ln(-q^2)\left\{\frac{21}{120}q^4 I_{\alpha\beta\gamma\delta} + \frac{23}{120}q^4\eta_{\alpha\beta}\eta_{\gamma\delta} - \frac{23}{120}(\eta_{\alpha\beta}q_\gamma q_\delta + \eta_{\gamma\delta}q_\alpha q_\beta) + \frac{21}{240}(q_\alpha q_\delta \eta_{\beta\gamma} + q_\alpha q_\gamma \eta_{\beta\delta} + q_\beta q_\gamma \eta_{\alpha\delta} + q_\beta q_\delta \eta_{\alpha\gamma}) + \frac{11}{30}q^\alpha q^\beta q^\gamma q^\delta\right\} + (\text{nonlogs}) .$$

$$\left(I_{\alpha\beta,\gamma\delta} \equiv \frac{1}{2}(\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma}) \right)$$

When reconstructing the Gravitational Potential, an addition form will be needed

$$P_{\mu\nu,\alpha\beta}\Pi^{\alpha\beta,\gamma\delta}P_{\gamma\delta,\rho\sigma} = \frac{\kappa^2 q^4}{32\pi^2}\left[\frac{21}{120}(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\nu\rho}\eta_{\mu\sigma}) + \frac{1}{120}\eta_{\mu\nu}\eta_{\rho\sigma}\right][-\ln(-q^2)] + \cdots$$

$$\left(P_{\mu\nu,\alpha\beta} = \frac{1}{2}[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\nu\alpha}\eta_{\mu\beta} - \eta_{\mu\nu}\eta_{\alpha\beta}] \right)$$

terms involving q_μ, q_ρ, etc. can be dropped since q_μ contracted with the vertex function gives a vanishing result.

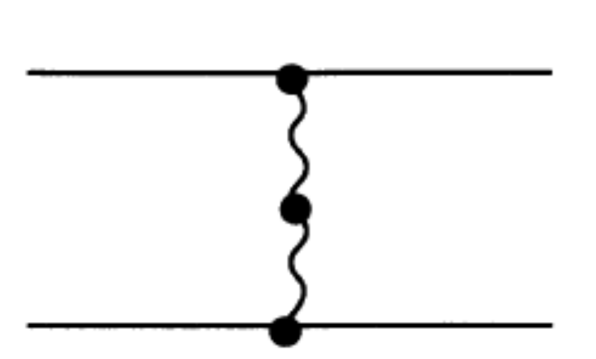
$$q_\mu \tau^{\mu\nu}_{\alpha\beta,\gamma\delta} = 0$$

J.F. Donoghue,
PRD50 (1994), 3874,
9512024 [gr-qc]

J.F. Donoghue,
PRD50 (1994), 3874,
9512024 [gr-qc]

$$-\frac{\kappa^2}{4}\frac{1}{2m_1}V^{(1)}_{\mu\nu}(q)\left[iD^{\mu\nu,\alpha\beta}(q)+iD^{\mu\nu,\rho\sigma}i\Pi_{\rho\sigma,\eta\lambda}iD^{\eta\lambda,\alpha\beta}\right]V_{\alpha\beta}(q)\frac{1}{2m_2}$$

$$\approx 4\pi G m_1 m_2\left[\frac{i}{\mathbf{q}^2}-\frac{i\kappa^2}{32\pi^2}\left[-\frac{127}{60}\ln(\mathbf{q}^2)+\frac{\pi^2(m_1+m_2)}{2\sqrt{\mathbf{q}^2}}\right]+\text{const}\right]$$



1-loop weak quantum gravity corrections to Newtonian potential between masses m_1 , m_2

$$V(r)=-\frac{Gm_1m_2}{r}\left[1-\frac{G(m_1+m_2)}{rc^2}-\frac{127}{30\pi^2}\frac{G\hbar}{r^2c^3}\right]$$

Part II

Other applications of weak Gravity as Effective Field Theory

(i) Axions and Gravity → Gravitational anomalies

→ Chern-Simons (CS) Gravity + chiral gravitational waves

→ gravitational-anomaly-Condensate-induced Inflation

(ii) Axions clouds in Rotating Black Holes

→ Superradiance Phenomenon

→ production of Squeezed entangled gravitons in gravitational waves

NB: Conventions of the Part II follow those in the following works: (-, +, +, +) in contrast to those of Donoghue in Part I (of course physics is non sensitive to this !)

(i) GravitationaL-CS-Condensate induced Inflation:
Dorlis, Vlachos, NEM
PRD 110 (2024), 063512;
Universe 11 (2025), 15

(ii) Squeezed entangled gravitons from rotating Black Holes:
Dorlis, Sarkar, Vlachos, NEM
PRL135 (2025), 151501;
PRD 113 (2026), 026023;
e-Print: 2605.14797 [gr-qc]
(IJMPD, 3rd Award 2026 GRF Essays on Gravitation)

Other applications of weak Gravity as Effective Field Theory

(i) Axions and Gravity → Gravitati... ...es

→ Chern-Simons (CS) ... ...waves

→ gravi... ...on

... Black Holes

...iance Phenomenon

To compute Chiral Grav. Wave Condensates
and graviton Squeezing we shall use:
Chern-Simons Gravity as weak-gravitational EFT
+ Canonical Quantization of gravitons

The latter are going a bit beyond EFT in a way we shall discuss

→ production of Squeezed entangled gravitons in gravitational waves

Other applications of weak Gravity as Effective Field Theory

(i) Axions and Gravity → Gravitational anomalies

→ Chern-Simons (CS) Gravity + chiral gravitational waves

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Other applications of weak Gravity as Effective Field Theory

$\boxed{\text{(i) Axions and Gravity}}$ → Gravitational anomalies

→ Chern-Simons (CS) Gravity + chiral gravitational waves

→ gravitational-anomaly-Condensate-induced Inflation

(i) Why Axions?

Because they elegantly solve two of the universe's greatest mysteries at once: **the missing dark matter** and the unexplained symmetry of the strong nuclear force (**strong CP problem**): Standard Model allows for Charge-Parity (CP) violation → if this were happening neutrons would have a dipole moment → axion as a dynamical field was invented to cancel such contributions by **tuning dynamically the CP violation** in the strong sector to **0.**

Axions occur naturally in the spectra of string-theory models

Particle Dark Matter

DARK MATTER (DM):

CURRENT EVIDENCE

Arguments in Favour

TYPES OF DM: hot, warm, cold

ASTROPHYSICAL CONSTRAINTS

(MODEL INDEPENDENT)

INDIRECT SEARCHES:

collider (LHC & beyond) searches

photons, neutrinos,

matter-antimatter asymmetries

(electron-positron, proton-antiproton)

THEORETICAL SCENARIOS

SUPERSYMMETRY ***neutralino***

SUPERGRAVITY ***gravitino*** ***(if sufficiently light)***

AXIONS *(standard QCD or stringy)*

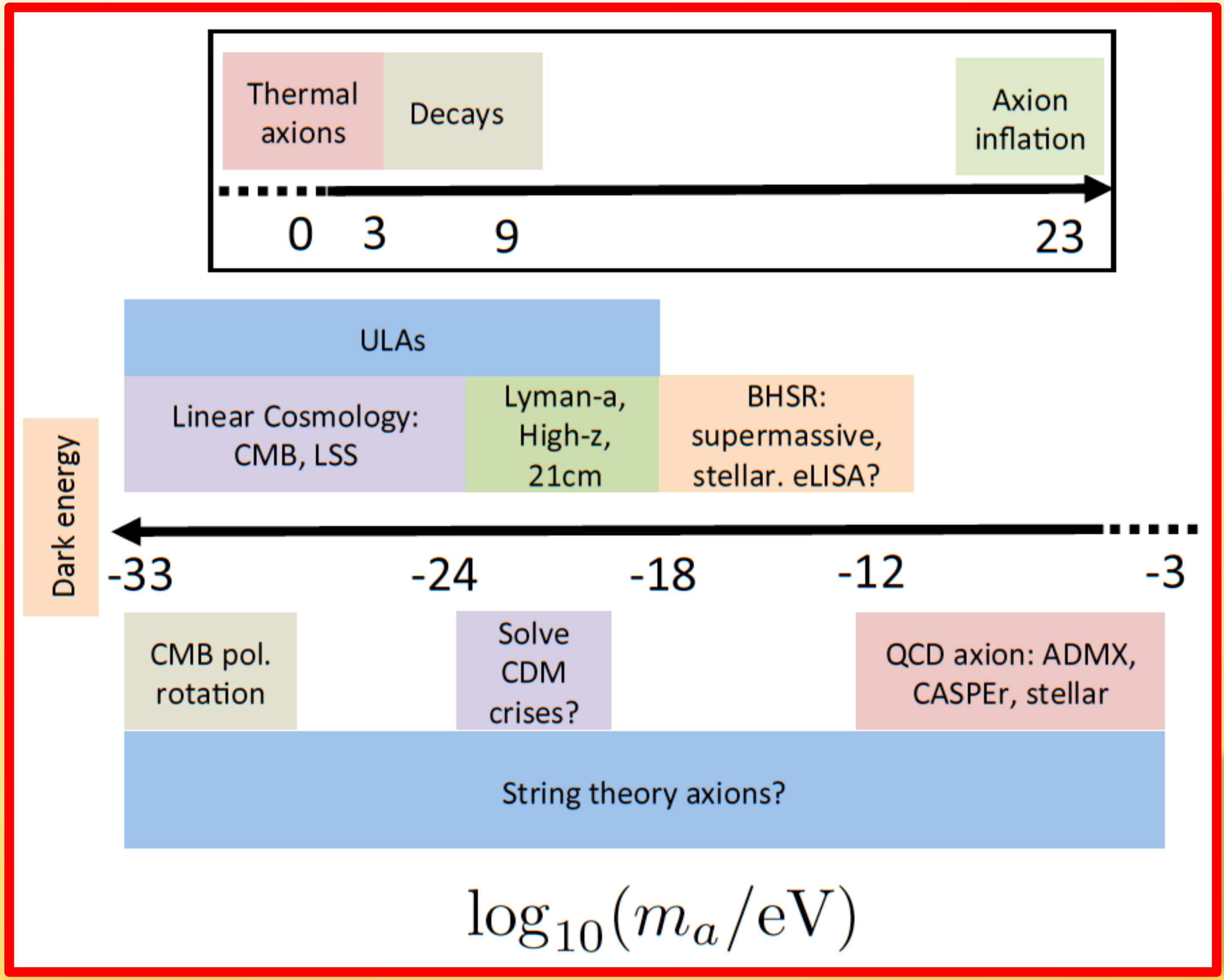
STERILE NEUTRINOS

…

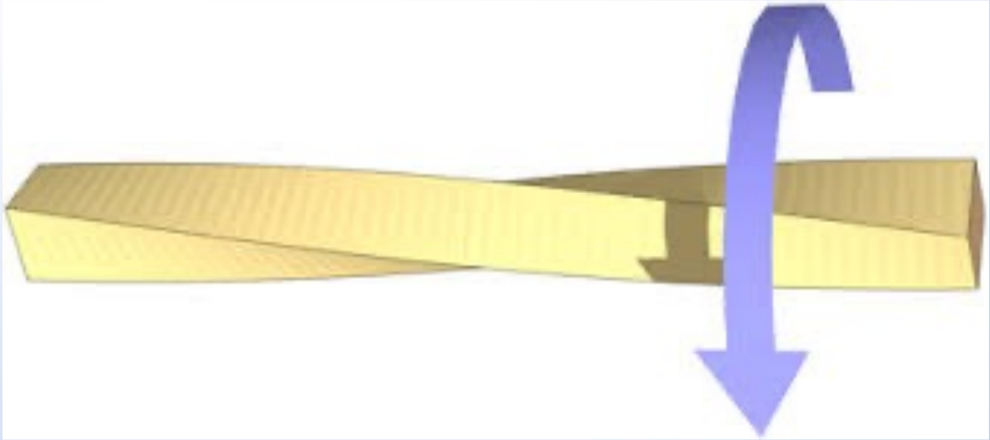
Axion Cosmology

D.J.E. Marsh,
Phys. Rept. 643, 1 (2016)
[arXiv:1510.07633 [astro-ph.CO]].

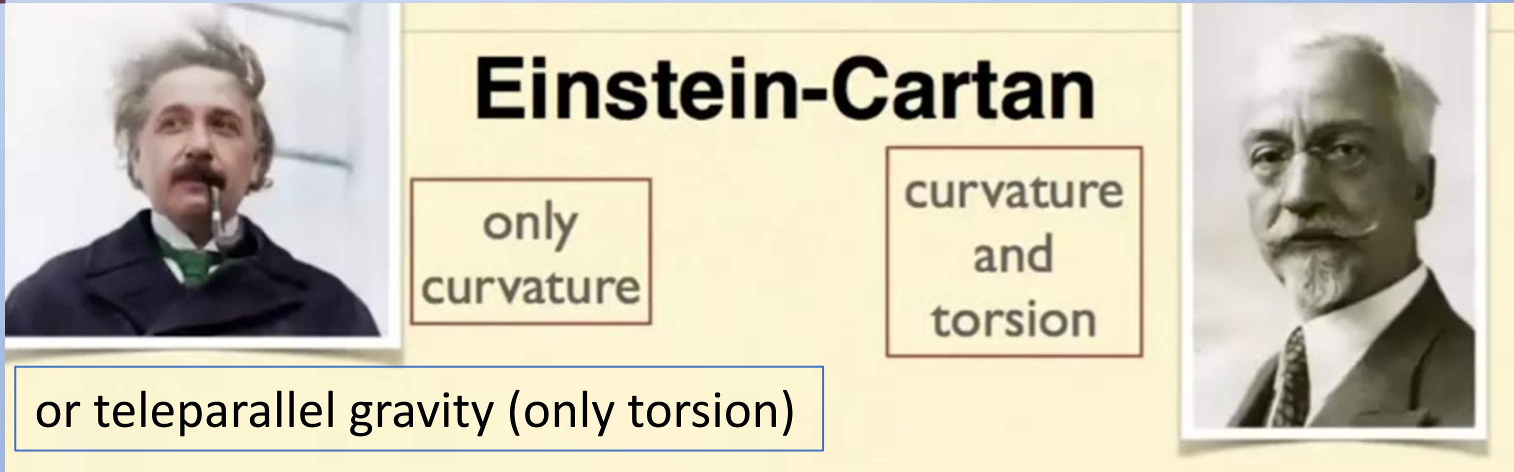
Cosmological Constraints & probes of axions



A Geometric Origin of axions ?

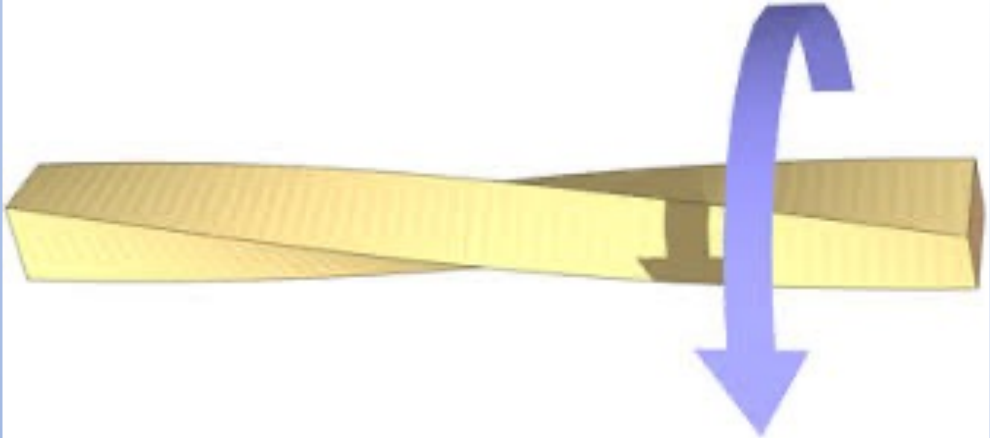


Torsion in spacetime?



or teleparallel gravity (only torsion)

A Geometric Origin of axions ?



Torsion in spacetime?

$$T^a_{\mu\nu} = e^a_\lambda\left(\Gamma^\lambda_{\nu\mu} - \Gamma^\lambda_{\mu\nu}\right) = -2e^a_\lambda\Gamma^\lambda_{[\mu\nu]}$$

Non-symmetric (lower indices) of Christoffel Sumbols

In (3+1) dimensional spacetimes

$$T_{[\mu\nu\rho]} \propto \varepsilon_{\mu\nu\rho\sigma}\,\partial^\sigma b(x)$$

Totally antisymm. Torsion component

Axion-like Particle (ALP) (initially Massless)

Covariant Levi-Civita Tensor density

Integrating out quantum torsion in Path Integral of (quantum) gravity models with torsion e.g. Einstein-Cartan Theory, or strings

→ dynamical ALP d.o.f.

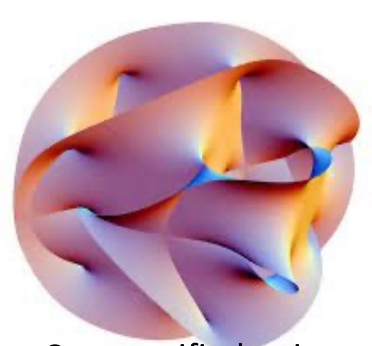
Ways of b(x) field to acquire Mass
→ Role as DM component of geometric origin

String-inspired gravitational theories with torsion and anomalies

Duncan, Kaloper, Olive (1992)

Massless gravitational (bosonic) string multiplet:

$$g_{\mu\nu} = g_{\nu\mu}\,, \quad \text{spin} = 2 \quad (\text{graviton})$$
$$\Phi\,, \quad \text{spin} = 0 \quad (\text{dilaton})\,,$$
$$B_{\mu\nu} = -B_{\nu\mu}\,, \quad \text{spin} = 1 \quad (\text{Kalb} - \text{Ramond (KR) field})$$



Compactified strings

Gauge symmetry in closed string sector $B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_\mu \theta_\nu - \partial_\nu \theta_\mu$

Effective target-spacetime gravitational action depends on the field strength : $H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$

String theory: Green-Schwarz mechanism for anomaly cancellation:

$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} + (\alpha'/\kappa)\,(\Omega_{3L\mu\nu\rho} - \Omega_{3Y\mu\nu\rho})$$

Chern-Simons terms

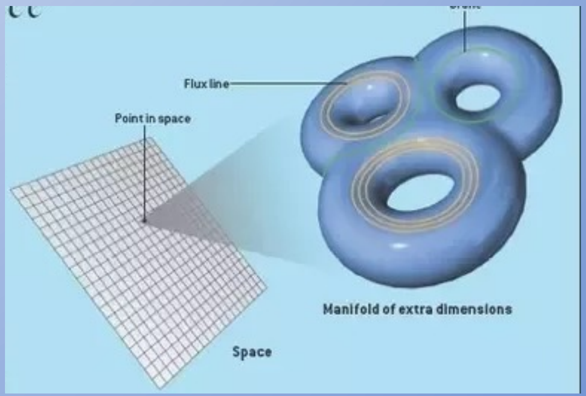
Gravitational **gauge**

α' = Regge slope = M_s^{-2}

$\kappa^2 = 8\pi G$ = 4d grav. constant

$$\Omega_{3L} = \omega^a{}_c \wedge \mathbf{d}\omega^c{}_a + \frac{2}{3}\omega^a{}_c \wedge \omega^c{}_d \wedge \omega^d{}_a$$

$$\Omega_{3Y} = \mathbf{A} \wedge \mathbf{dA} + \mathbf{A} \wedge \mathbf{A} \wedge \mathbf{A}$$



String-inspired gravitational theories with torsion and anomalies

Duncan, Kaloper, Olive (1992)

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$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} + (\alpha'/\kappa)\,(\Omega_{3L\mu\nu\rho} - \Omega_{3Y\mu\nu\rho})$$

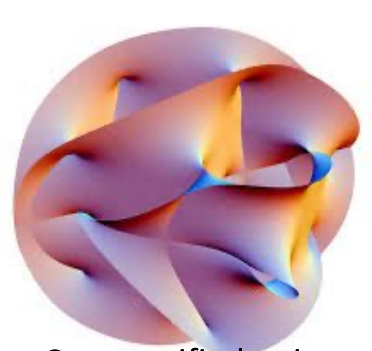
String effective action (lowest order in Regge slope)

$$S_B = \int d^4x \sqrt{-g}\Big(\frac{1}{2\kappa^2} R + \frac{1}{6}\mathcal{H}_{\lambda\mu\nu}\mathcal{H}^{\lambda\mu\nu} + \dots\Big).$$

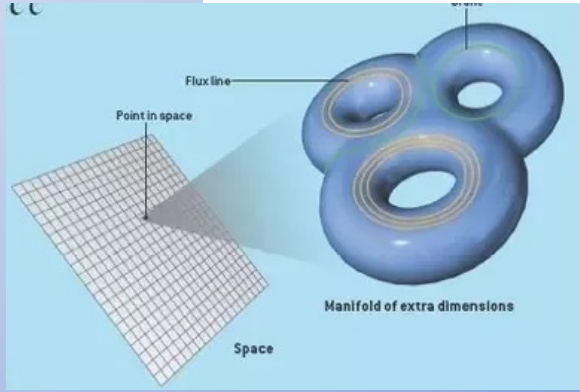
Totally antisymmetric torsion

$$\overline{R}(\overline{\Gamma})$$

$$\overline{\Gamma}^{\rho}_{\mu\nu} = \Gamma^{\rho}_{\mu\nu} + \frac{\kappa}{\sqrt{3}}\mathcal{H}^{\rho}_{\mu\nu} \neq \overline{\Gamma}^{\rho}_{\nu\mu}$$



Compactified strings



Torsion → axion-like d.o.f. (as in CONTORTED QED)

String-model Kalb-Ramond (KR)m independent axion

Svrcek-Witten

String-inspired gravitational theories with torsion and anomalies

Duncan, Kaloper, Olive (1992)

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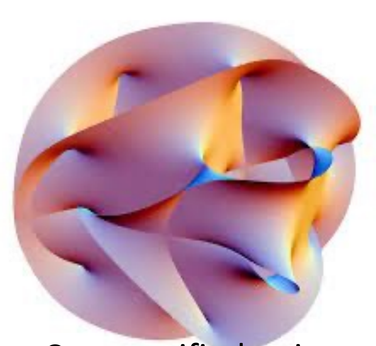
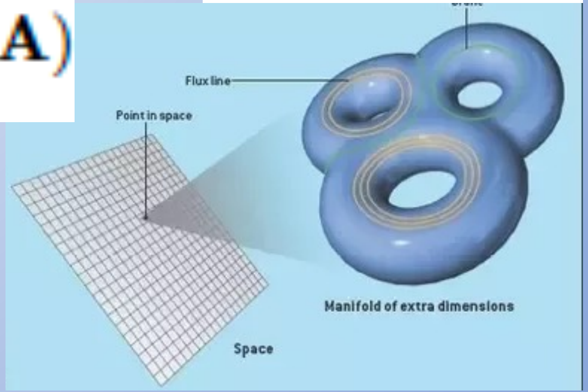
$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} + (\alpha'/\kappa)\,(\Omega_{3L\mu\nu\rho} - \Omega_{3Y\mu\nu\rho})$$



Compactified strings

Bianchi identity constraint

$$\varepsilon_{abc}{}^{\mu}\,\mathcal{H}^{abc}{}_{;\mu} = \frac{\alpha'}{32\,\kappa}\sqrt{-g}\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) \equiv \sqrt{-g}\,\mathcal{G}(\omega, \mathbf{A})$$



$$\widetilde{R}_{\alpha\beta\gamma\delta} = \frac{1}{2}R_{\alpha\beta}{}^{\rho\sigma}\,\varepsilon_{\rho\sigma\gamma\delta}$$

$$\widetilde{\mathbf{F}}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}\,\mathbf{F}^{\alpha\beta}$$

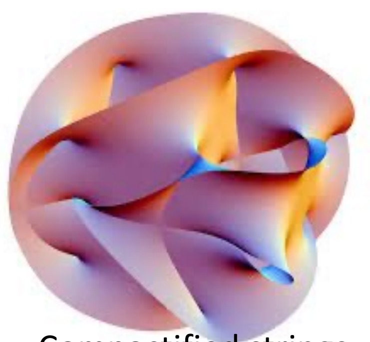
Dual tenors

String-inspired gravitational theories with torsion and anomalies

Duncan, Kaloper, Olive (1992)

String theory: Green-Schwarz mechanism for anomaly cancellation:

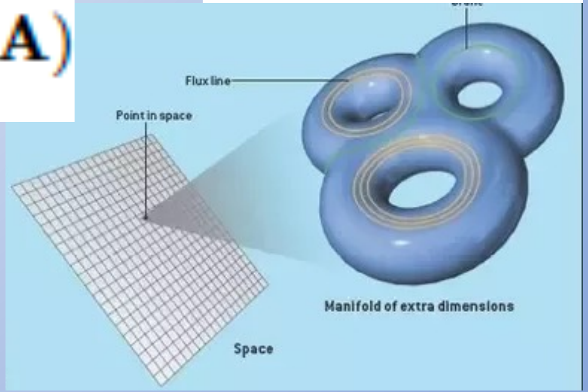
$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} + (\alpha'/\kappa)\,(\Omega_{3L\mu\nu\rho} - \Omega_{3Y\mu\nu\rho})$$



Compactified strings

Bianchi identity constraint

$$\varepsilon_{abc}{}^{\mu}\,\mathcal{H}^{abc}{}_{;\mu} = \frac{\alpha'}{32\,\kappa}\sqrt{-g}\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) \equiv \sqrt{-g}\,\mathcal{G}(\omega, \mathbf{A})$$



Implementation via axion-like Lagrange multiplier field b(x)

$$\Pi_x\,\delta\Big(\varepsilon^{\mu\nu\rho\sigma}\,\mathcal{H}_{\nu\rho\sigma}(x)_{;\mu} - \mathcal{G}(\omega, \mathbf{A})\Big) \Rightarrow$$

$$\int \mathcal{D}b\,\exp\left[i\int d^4x\sqrt{-g}\,\frac{1}{\sqrt{3}}\,b(x)\Big(\varepsilon^{\mu\nu\rho\sigma}\,\mathcal{H}_{\nu\rho\sigma}(x)_{;\mu} - \mathcal{G}(\omega, \mathbf{A})\Big)\right]$$

$$= \int \mathcal{D}b\,\exp\left[-i\int d^4x\sqrt{-g}\,\Big(\partial^{\mu}b(x)\,\frac{1}{\sqrt{3}}\,\epsilon_{\mu\nu\rho\sigma}\,\mathcal{H}^{\nu\rho\sigma} + \frac{b(x)}{\sqrt{3}}\,\mathcal{G}(\omega, \mathbf{A})\Big)\right]$$

Sarkar, NEM
Basilakos, Solà, NEM

String-inspired gravitational theories with torsion and anomalies

Duncan, Kaloper, Olive (1992)

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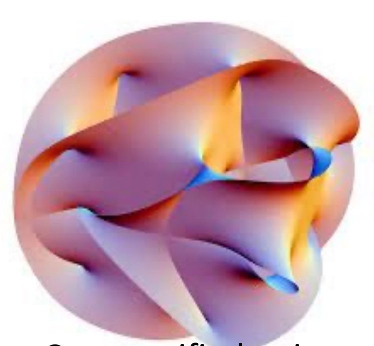

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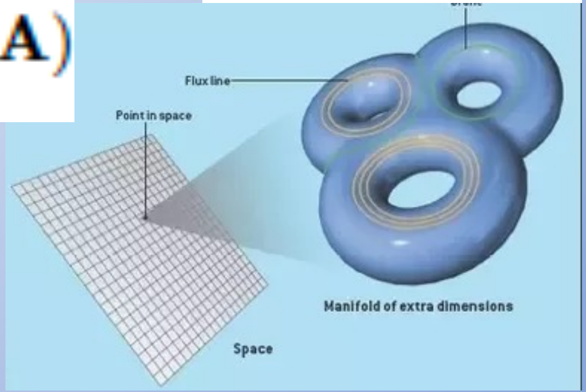

Implementation via axion-like Lagrange multiplier field b(x)

Integration of non-propagating H field

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[\; \frac{1}{2\kappa^2} R - \frac{1}{2}\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\, b(x)\left(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\right) + \dots\Big]$$

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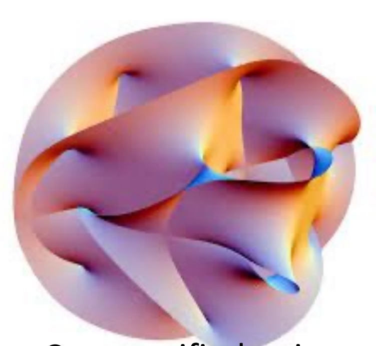

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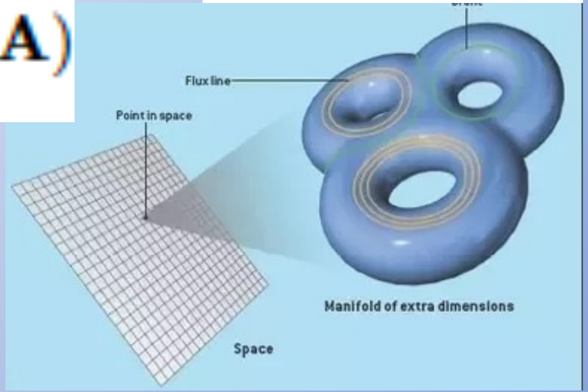

NEM,
+ Basilakos, Solà,
Spanos, Stamou,
Dorlis, Vlachos,
Vyros

Massive axions through
Non-Abelian gauge group
Instantons

$$S^{\rm eff}_B = \int d^4x\sqrt{-g}\Big[\frac{1}{2\kappa^2}R - \frac{1}{2}\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\,b(x)\left(R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\widetilde{F}^{\mu\nu}\right) + \dots\Big]$$

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GEOMETRIC ORIGIN OF AXION DM

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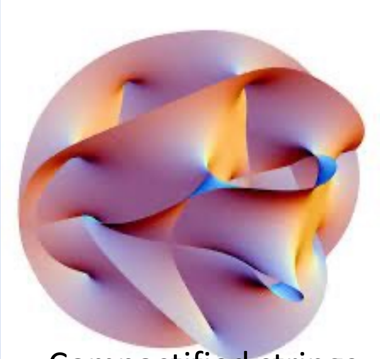

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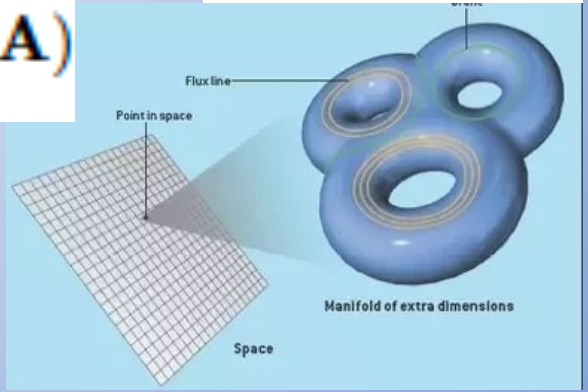

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String KR axions, Condensates & Inflation

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Fixed coupling
for the string-model
independent axion *b*(x)

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Basilakos, NEM, Solà-Peracaula
+ Sarkar, Saridakis, Tzerefos, Papanikolaou,
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Dorlis, Vlachos, Vyros (**2019 - 2025)**

In the early Universe, the basic assumption of the model is that metric and axion are the and the gravitational anomaly interaction constitute the only non-trivial dynamical content → $F_{\mu\nu} = 0$

Massless axion b(x)

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This is the basic Lagrangian of the **Stringy Running Vacuum Model of Cosmology (StRVM)**

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Massless axion b(x)

This is a Chern-Simons Gravity / Cosmology

Jackiw & Pi,
Phys.Rev.D 68 (2003) 104012

NB: By **equivalence** of **perturbative S-matrix** and **string-amplitude calculations** under the requirements of **unitarity** and **torsion interpretation** of **Kalb-Ramond field strength**, one arrives at a more general effective action (after compactification to (3+1) dim)

G. Panagopoulos, NEM
Universe 12 (2026) 3
e-Print: 2602.16076 [gr-qc]

$$S_E = \int \left[\frac{1}{2q^2} R - \frac{1}{2}\partial_\mu b \partial^\mu b + \frac{\epsilon}{8} b R_{\mu\nu\lambda\rho}\tilde{R}^{\mu\nu\lambda\rho} - \frac{\lambda}{2}\partial_\mu b \partial_\nu b R^{\mu\nu}\right]\sqrt{-g}\, d^4x,$$

$$\lambda = \alpha' \left(\frac{1}{14} - \frac{11}{63\sqrt{3}}\right) < 0\,,$$

$$\frac{1}{2q^2} = \frac{1}{2\kappa^2}\left[1 - \frac{\kappa^2}{\alpha'}\frac{\lambda}{\alpha'}\right],$$

$$\epsilon = \frac{\alpha'}{\kappa}\frac{\sqrt{2}}{12}.$$

cf. normalization
$\alpha' \to 3^{-1/2}\,\alpha'$
to recover results of **Basilakos, Solà, Dorlis, Vlachos, Vyros, NEM**

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Negligible quantitative changes, compare to StRVM

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$$S_B^{\rm eff} = \int d^4x \sqrt{-g}\Big[\frac{1}{2\kappa^2} R - \frac{1}{2}\partial_\mu b\, \partial^\mu b + \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\, b(x)\,\Big(R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\, \widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

Primordial string Universe Gravitational Waves (GW): e.g. from collapse of (rotating) primordial black holes (PBH) sourced by Torsion-induced axion field *b(x)*

$$\Box b \propto R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}$$

Lyth, Rodriguez, Quimbay
Alexander, Peskin, Sheikh-Jabbari
Dorlis, Vlachos, NEM

can induce **condensates** of gravitational Chern-Simons terms $\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle$

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can induce **condensates** of gravitational Chern-Simons

Inflation of RVM type

Condensates lead to linear axion potentials

$$V(b) \ni b(x)\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle$$

Dorlis, Vlachos, NEM
PRD 110 (2024), 063512;
Universe 11 (2025), 15

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can induce **condensates** of gravitational Chern-Simons terms $\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle$

First: Compute using weak (perturbative) Quantum gravity techniques With **chiral GW** perturbation modes

Condensates lead to linear axion potentials

$$V(b) \ni b(x)\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle$$

Lyth, Rodriguez, Quimbay
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Alexander, Peskin,
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Estimating the Condensate

Treat Chern-Simons Gravity as an EFT
+ canonical quantization techniques for
weak chiral gravitational waves

Dorlis, NEM, Vlachos
PRD 110 (2024), 063512;
Universe 11 (2025), 15

Improvement on approximations made in previous works

Alexander, Peskin, Sheikh –Jabbari
Lyth, Rodriguez, Quimbay

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Dorlis, NEM, Vlachos
PRD 110 (2024), 063512;
Universe 11 (2025), 15

Dorlis, NEM, Vlachos, PRD 110 (2024), 063512, arXive:2403.09005 [gr-qc]

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \frac{1}{2}(\partial_\mu b)(\partial^\mu b) - A\, b\, R_{CS} \right] ,$$

$$\boxed{R_{CS} = \frac{1}{2} R^\mu{}_{\nu\rho\sigma} \widetilde{R}^\nu{}_\mu{}^{\rho\sigma}}$$

$$A = \sqrt{\frac{2}{3}} \frac{\alpha'}{48\kappa}$$

$$2\langle R_{CS} \rangle = -\langle R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} \rangle = -\int DbDg_{\mu\nu} e^{-S^{\text{eff}}} R_{\mu\nu\rho\sigma} \widetilde{R}^{\mu\nu\rho\sigma} = \text{constant}$$

Within weak Chiral GW quantum flcts approximation about FLRW background!

$$\widehat{g}_{\mu\nu} n = g_{\mu\nu}^{\text{FLRW}(0)} + \kappa \widehat{h}_{\mu\nu}$$

$$\langle \dots \rangle = \langle 0| \dots |0\rangle$$

Bunch-Davies vacuum

tensor perturbation of the FLRW

(assume spatially flat Universe)

$$ds^2 = -dt^2 + \alpha^2(t)(\delta_{ij} + h_{ij})dx^i dx^j$$

$$h_{ij} = h_+\epsilon_{ij}^{(+)} + h_\times\epsilon_{ij}^{(\times)}$$

Polarization tensors

$$\epsilon_{ij}^{(+)} = [e_1(\vec{k})]_i[e_1(\vec{k})]_j - [e_2(\vec{k})]_i[e_2(\vec{k})]_j,$$

$$\epsilon_{ij}^{(\times)} = [e_1(\vec{k})]_i[e_2(\vec{k})]_j + [e_1(\vec{k})]_j[e_2(\vec{k})]_i,$$

$$(e_1(\vec{k}), e_2(\vec{k}), e_3(\vec{k})) \qquad e_3(\vec{k}) = \vec{k}/|\vec{k}|$$

right-handed orthogonal triad of unit vectors

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right-handed orthogonal triad of unit vectors

choose

$$e_1 = (1, 0, 0), \; e_2 = (0, 1, 0), \; e_3 = (0, 0, 1)$$

$$[h_{ij}] = \begin{bmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$h = h^i_i = 0 \qquad h_{+,\times} = h_{+,\times}(t, z)$$

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Expansion in terms of Left (L), rght (R) polarizatrions $$h_{ij}(t,\vec{x}) = h_L \epsilon_{ij}^{(L)} + h_R \epsilon_{ij}^{(R)} = \sum_{\lambda=L,R} h_\lambda(t,\vec{x})\epsilon_{ij}^{(\lambda)}$$

$$[\epsilon_{ij}^{(R)}] = \frac{1}{\sqrt{2}}([\epsilon_{ij}^{(+)}] + i[\epsilon_{ij}^{(\times)}]) = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & i & 0 \\ i & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = [\epsilon_{ij}^{(L)}]^\dagger , \qquad \epsilon_{ij}^{*(\lambda)}\epsilon^{ij}_{(\lambda')} = 2\delta_{\lambda\lambda'} .$$

$$R_{\rm CS} = \frac{1}{2} R^\mu_{\nu\rho\sigma}\tilde{R}^{\nu\rho\sigma}_\mu = \frac{2i}{\alpha^3}[(\partial_z^2 h_L \partial_z\partial_t h_R + \alpha^2\partial_t^2 h_L \partial_z\partial_t h_R + \alpha\dot{\alpha}\partial_t h_L \partial_z\partial_t h_R) - L \leftrightarrow R] + \mathcal{O}(h^4).$$

Non-trivial for chiral GW perturbations $h_L \neq h_R$

Gravitational waves in Chern-Simons gravity

$$\boxed{S = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} - \frac{1}{2}(\partial_\mu b)(\partial^\mu b) - AbR_{CS}\right]}$$

$$A = \sqrt{\frac{2}{3}}\frac{\alpha'}{48\kappa}$$

$$\boxed{R_{CS} = \frac{1}{2}R^\mu{}_{\nu\rho\sigma}\tilde{R}^\nu{}_\mu{}^{\rho\sigma}}$$

Equations of motion

$$\boxed{ds^2 = -dt^2 + \alpha^2(t)(\delta_{ij} + h_{ij})dx^i dx^j}$$

$$G_{\mu\nu} = \kappa^2 T^{(b)}_{\mu\nu} + 4\kappa^2 AC_{\mu\nu},$$

$$T^{(b)}_{\mu\nu} = \nabla_\mu b\nabla_\nu b - \frac{1}{2}g_{\mu\nu}(\nabla b)^2$$

Cotton

$$\Box b = AR_{CS},$$

$$C_{\mu\nu} = -\frac{1}{2}\nabla^\alpha[(\nabla^\beta b)\tilde{R}_{\alpha\mu\beta\nu} + (\nabla^\beta b)\tilde{R}_{\alpha\nu\beta\mu}].$$

Linearized gravitational eqs wrt $h_{\times,+}$

$$\Box = -\partial_t^2 - 3\frac{\dot{\alpha}}{\alpha}\partial_t + \frac{1}{\alpha^2}\partial_z^2$$

$$\boxed{\Box h_{R,L} = \pm\frac{4iA\kappa^2}{\alpha^2}(2\dot{\alpha}\dot{b} + \alpha\ddot{b})\partial_t\partial_z h_{R,L} \pm \frac{4iA\kappa^2}{\alpha}\dot{b}\partial_t^2\partial_z h_{R,L} \mp \frac{4iA\kappa^2}{\alpha^3}\dot{b}\partial_z^3 h_{R,L}}$$

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Linearized gravitational eqs wrt $h_{x,+}$ in conformal time

$$\boxed{dt = a\,d\eta}$$

$$\boxed{h''_\lambda + 2\frac{a'}{a}h'_\lambda - \partial_z^2 h_\lambda = -l_\lambda\frac{4iA\kappa^2}{a^2}\partial_z(b''h'_\lambda + b'h''_\lambda - b'\partial_z^2 h_\lambda)}$$

$\lambda = R, L$

$l_R = +1,\ \ l_L = -1$

Strategy on the Quantum Condensate Estimate

- Pass onto Fourier Trnsf : $$h_\lambda(\eta,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}}\tilde{h}_{\lambda,\vec{k}}(\eta),$$

- Linearised eqs of $\tilde{h}_{\lambda,\vec{k}}(\eta)$: $$\tilde{h}''_{\lambda,\vec{k}} + 2\frac{\alpha'}{\alpha}\tilde{h}'_{\lambda,\vec{k}} + k^2\tilde{h}_{\lambda,\vec{k}} = l_\lambda l_{\vec{k}}\frac{4kA\kappa^2}{\alpha^2}(b''\tilde{h}'_{\lambda,\vec{k}} + b'\tilde{h}''_{\lambda,\vec{k}} + k^2 b'\tilde{h}_{\lambda,\vec{k}})$$

- Change of variables: $$\tilde{h}_{\lambda,\vec{k}}(\eta) = \kappa\frac{\tilde{\psi}_{\lambda,\vec{k}}(\eta)}{z_{\lambda,\vec{k}}(\eta)}$$ $$z_{\lambda,\vec{k}}(\eta) = \alpha\sqrt{1 - l_\lambda l_{\vec{k}} L_{\rm CS}(\eta)},$$ $$L_{\rm CS}(\eta) = k\xi, \qquad \xi = \frac{4Ab'\kappa^2}{\alpha^2}$$

- Linearised eqs of tensor perts. : $$\boxed{\tilde{\psi}''_{\lambda,\vec{k}} + \omega^2_{\lambda,\vec{k}}(\eta)\tilde{\psi}_{\lambda,\vec{k}} = 0, \qquad \lambda = L, R,}$$ $$\tilde{\psi}^*_{L,-\vec{k}}(\eta) = \tilde{\psi}_{R,\vec{k}}(\eta) \qquad \omega^2_{\lambda,\vec{k}}(\eta) = k^2 - \frac{z''_{\lambda,\vec{k}}(\eta)}{z_{\lambda,\vec{k}}(\eta)}.$$

Strategy on the Quantum Condensate Estimate

- Pass onto Fourier Trnsf :

$$h_\lambda(\eta,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \tilde{h}_{\lambda,\vec{k}}(\eta),$$

Map the system into that of two (quantum) scalar fields:

$$\phi(\eta,\vec{x}) = \psi_L(\eta,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \tilde{\psi}_{L,\vec{k}}(\eta), \quad \tilde{\phi}_{\vec{k}} = \tilde{\psi}_{L,\vec{k}},$$

$$\phi^*(\eta,\vec{x}) = \psi_R(\eta,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \tilde{\psi}_{R,\vec{k}}(\eta), \quad \tilde{\phi}^*_{-\vec{k}} = \tilde{\psi}_{R,\vec{k}},$$

$$\ldots_{\vec{k}} = l_\lambda l_{\vec{k}} \frac{4kA\kappa^2}{\alpha^2}\left(b''\tilde{h}'_{\lambda,\vec{k}} + b'\tilde{h}''_{\lambda,\vec{k}} + k^2 b'\tilde{h}_{\lambda,\vec{k}}\right)$$

$$z_{\lambda,\vec{k}}(\eta) = \alpha\sqrt{1 - l_\lambda l_{\vec{k}} L_{\rm CS}(\eta)},$$

$$L_{\rm CS}(\eta) = k\xi, \qquad \xi = \frac{4Ab'\kappa^2}{\alpha^2}$$

- Linearised eqs of tensor perts. :

$$\tilde{\psi}''_{\lambda,\vec{k}} + \omega^2_{\lambda,\vec{k}}(\eta)\tilde{\psi}_{\lambda,\vec{k}} = 0, \qquad \lambda = L, R,$$

$$\tilde{\psi}^*_{L,-\vec{k}}(\eta) = \tilde{\psi}_{R,\vec{k}}(\eta)$$

$$\omega^2_{\lambda,\vec{k}}(\eta) = k^2 - \frac{z''_{\lambda,\vec{k}}(\eta)}{z_{\lambda,\vec{k}}(\eta)}.$$

Strategy on the Quantum Condensate Estimate

❖ Pass onto Fourier Trnsf :

$$h_\lambda(\eta,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \tilde{h}_{\lambda,\vec{k}}(\eta),$$

Map the system into that of two (quantum) scalar fields:

$$\phi(\eta,\vec{x}) = \psi_L(\eta,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \tilde{\psi}_{L,\vec{k}}(\eta), \quad \tilde{\phi}_{\vec{k}} = \tilde{\psi}_{L,\vec{k}},$$

$$\phi^*(\eta,\vec{x}) = \psi_R(\eta,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\vec{k}\cdot\vec{x}} \tilde{\psi}_{R,\vec{k}}(\eta), \quad \tilde{\phi}^*_{-\vec{k}} = \tilde{\psi}_{R,\vec{k}},$$

Introduce two sets of creation-annihilation operators of scalar fields:

$$\alpha^\pm_{\vec{k}} \text{ and } b^\pm_{\vec{k}} \qquad (\alpha^-_{\vec{k}})^\dagger = \alpha^+_{\vec{k}} \qquad (b^-_{\vec{k}})^\dagger = b^+_{\vec{k}}$$

Canonical commutators:

$$[\hat{\alpha}^-_{\vec{k}}, \hat{\alpha}^+_{\vec{k}'}] = [\hat{b}^-_{\vec{k}}, \hat{b}^+_{\vec{k}'}] = \delta^{(3)}(\vec{k}-\vec{k}')$$

❖ Linearised eqs of tensor perts. :

$$\tilde{\psi}''_{\lambda,\vec{k}} + \omega^2_{\lambda,\vec{k}}(\eta)\tilde{\psi}_{\lambda,\vec{k}} = 0, \qquad \lambda = L, R,$$

$$\tilde{\psi}^*_{L,-\vec{k}}(\eta) = \tilde{\psi}_{R,\vec{k}}(\eta)$$

$$\omega^2_{\lambda,\vec{k}}(\eta) = k^2 - \frac{z''_{\lambda,\vec{k}}(\eta)}{z_{\lambda,\vec{k}}(\eta)}.$$

Mode functions:

$$\{\tilde{v}_{\vec{k}}, \tilde{v}^*_{\vec{k}}\}$$

$$\hat{\tilde{\phi}}_{\vec{k}}(\eta) = \tilde{v}_{\vec{k}}\hat{\alpha}^-_{\vec{k}} + v^*_{-\vec{k}}\hat{b}^+_{-\vec{k}},$$

$$\hat{\tilde{\phi}}^*_{-\vec{k}}(\eta) = v_{\vec{k}}\hat{b}^-_{\vec{k}} + \tilde{v}^*_{-\vec{k}}\hat{\alpha}^+_{-\vec{k}}.$$

conjugate momenta $\hat{\tilde{\pi}}_{\vec{k}} = -\hat{\tilde{\phi}}^{*\prime}_{\vec{k}}$ $\Rightarrow$ $\boxed{[\hat{\tilde{\phi}}_{\vec{k}}(\eta), \hat{\tilde{\pi}}_{\vec{k}'}(\eta)] = i\delta(\vec{k}-\vec{k}')}$

$$\boxed{\begin{aligned} \tilde{v}_{\vec{k}}\tilde{v}^{*\prime}_{\vec{k}} - \tilde{v}^*_{\vec{k}}\tilde{v}'_{\vec{k}} &= -i, \\ v_{\vec{k}}v^{*\prime}_{\vec{k}} - v^*_{\vec{k}}v'_{\vec{k}} &= -i. \end{aligned}}$$

Chiral GW tensor perturbations (quantized)

$$\boxed{\begin{aligned} \hat{\tilde{h}}_{L,\vec{k}} &= u_{L,\vec{k}}\hat{\alpha}^-_{\vec{k}} + u^*_{R,-\vec{k}}\hat{b}^+_{-\vec{k}}, \\ \hat{\tilde{h}}_{R,\vec{k}} &= u_{R,\vec{k}}\hat{b}^-_{\vec{k}} + u^*_{L,-\vec{k}}\hat{\alpha}^+_{-\vec{k}}, \end{aligned}}$$

$$u_{L,\vec{k}} = \kappa\frac{\tilde{v}_{\vec{k}}}{z_{L,\vec{k}}},$$

$$u_{R,\vec{k}} = \kappa\frac{v_{\vec{k}}}{z_{R,\vec{k}}}.$$

Correlators between the helicity basis L, R are non-trivial

$$\langle \tilde{h}_{R,\vec{k}_1} \tilde{h}_{L,\vec{k}_2} \rangle = u_{R,\vec{k}_1} u^*_{R,-\vec{k}_2} \delta(\vec{k}_1 + \vec{k}_2),$$

$$\langle \tilde{h}_{L,\vec{k}_1} \tilde{h}_{R,\vec{k}_2} \rangle = u_{L,\vec{k}_1} u^*_{L,-\vec{k}_2} \delta(\vec{k}_1 + \vec{k}_2).$$

And the gCS condensate at a given cosmological era reads formally

$$\langle R_{\rm CS} \rangle = \frac{2i}{\alpha^4} [\langle \partial_z^2 h_L \partial_z h'_R \rangle + \langle h''_L \partial_z h'_R \rangle - \langle \partial_z^2 h_R \partial_z h'_L \rangle - \langle h''_R \partial_z h'_L \rangle],$$

$$\boxed{\begin{aligned}\langle R_{\rm CS} \rangle = \frac{2}{\alpha^4} \int^{\alpha\mu} \frac{d^3\vec{k}}{(2\pi)^3} l_{\vec{k}} \big[& k^3 (u_{L,\vec{k}} u^{*\prime}_{L,\vec{k}} - u_{R,\vec{k}} u^{*\prime}_{R,\vec{k}}) \\ & + k (u''_{R,\vec{k}} u^{*\prime}_{R,\vec{k}} - u''_{L,\vec{k}} u^{*\prime}_{L,\vec{k}}) \big].\end{aligned}}$$

Correlators between the helicity basis L, R are non-trivial

$$\langle \tilde{h}_{R,\vec{k}_1} \tilde{h}_{L,\vec{k}_2} \rangle = u_{R,\vec{k}_1} u^*_{R,-\vec{k}_2} \delta(\vec{k}_1 + \vec{k}_2),$$

$$\langle \tilde{h}_{L,\vec{k}_1} \tilde{h}_{R,\vec{k}_2} \rangle = u_{L,\vec{k}_1} u^*_{L,-\vec{k}_2} \delta(\vec{k}_1 + \vec{k}_2).$$

And the gCS condensate at a given cosmological era reads formally

$$\langle R_{CS} \rangle = \frac{2i}{\alpha^4} [\langle \partial_z^2 h_L \partial_z h'_R \rangle + \langle h''_L \partial_z h'_R \rangle - \langle \partial_z^2 h_R \partial_z h'_L \rangle - \langle h''_R \partial_z h'_L \rangle],$$

$$\boxed{\langle R_{CS} \rangle = \frac{2}{\alpha^4} \int^{\alpha\mu} \frac{d^3\vec{k}}{(2\pi)^3} l_{\vec{k}} \left[k^3 (u_{L,\vec{k}} u^{*\prime}_{L,\vec{k}} - u_{R,\vec{k}} u^{*\prime}_{R,\vec{k}}) + k (u''_{R,\vec{k}} u^{*\prime}_{R,\vec{k}} - u''_{L,\vec{k}} u^{*\prime}_{L,\vec{k}}) \right].}$$

μ = UV cutoff of gravitons

Gravitational Anomaly Condensate and RVM Inflation

During inflation

H (Hubble) ≈ H_I = constant during inflation

$$\alpha(t) = \frac{1}{H_I\,\eta}, \quad \eta < 0$$

Cosmological Data

$$H_I \lesssim 10^{-5}\, M_{\rm Pl}$$

(= O(10^{13}) GeV)

Chiral GW eqs

$$\tilde\psi''_{\lambda,\vec k} + \omega^2_{\lambda,\vec k}(\eta)\tilde\psi_{\lambda,\vec k} = 0,$$

$|L_{CS}| \ll 1$

$$|L_{\rm CS}| = \tilde L_{\rm CS}\; k\eta$$

$$\tilde L_{\rm CS} = 4A\dot b \kappa^2 H_I \sim 10^{-11}$$

(cf. below for StRVM pheno)

max (x) = $k\,|\eta|$ = O(10^4)

$$L^{\max}_{\rm CS} \sim 10^{-7} \ll 1$$

$$\frac{d^2}{dx^2}\tilde\psi_{\lambda,\vec k} + \left[1 - \frac{2}{x^2}\left(1 - l_\lambda l_{\vec k}\frac{L_{CS}}{2} + \mathcal{O}(L^2_{CS})\right)\right]\tilde\psi_{\lambda,\vec k} = 0.$$

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Universe 11 (2025), 15

Gravitational Anomaly Condensate and RVM Inflation

During inflation

H (Hubb… inflation

$\alpha(t) =$

Chiral GV…

$\tilde{\psi}''_{\lambda,\vec{k}} + \omega$…

Plane-Wave approximate solutions

$$v_k = \frac{1}{\sqrt{2k}} e^{ik\eta},$$

$$v_k^* = \frac{1}{\sqrt{2k}} e^{-ik\eta};$$

$$\frac{d^2}{dx^2}\tilde{\psi}_{\lambda,\vec{k}} + \left[1 - \frac{2}{x^2}\left(1 - l_\lambda l_{\vec{k}} \frac{L_{CS}}{2} + \mathcal{O}(L_{CS}^2)\right)\right]\tilde{\psi}_{\lambda,\vec{k}} = 0$$

Cosmological Data

$$H_I \lesssim 10^{-5}\, M_{\rm Pl}$$

(= O(10^{13}) GeV)

$$|L_{CS}| = \tilde{L}_{CS}\ k\eta$$

$$\tilde{L}_{CS} = 4A\dot{b}\kappa^2 H_I \sim 10^{-11}$$

(cf. below for StRVM pheno)

max (x) = k | n | = O(10^4)

$$L_{CS}^{\max} \sim 10^{-7} \ll 1$$

Dorlis, Vlachos, NEM
PRD 110 (2024), 063512;
Universe 11 (2025), 15

Gravitational Anomaly Condensate and RVM Inflation

During inflation

$$z^I_{L,\vec{k}}(\eta) = -\frac{1}{H_I\eta}\sqrt{1 - l_{\vec{k}}4A\dot{b}H_I^2 k\kappa^2\eta},$$
$$z^I_{R,\vec{k}}(\eta) = -\frac{1}{H_I\eta}\sqrt{1 + l_{\vec{k}}4A\dot{b}H_I^2 k\kappa^2\eta}.$$

$\alpha(t) =$

Plane-Wave approximate solutions

$$v_k = \frac{1}{\sqrt{2k}}e^{ik\eta},$$
$$v_k^* = \frac{1}{\sqrt{2k}}e^{-ik\eta};$$

Chiral GW

$\tilde{\psi}''_{\lambda,\vec{k}} + \omega$

$$|L_{CS}| = \tilde{L}_{CS}\ k\eta$$
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$$\frac{d^2}{dx^2}\tilde{\psi}_{\lambda,\vec{k}} + \left[1 - \frac{2}{x^2}\left(1 - l_\lambda l_{\vec{k}}\frac{L_{CS}}{2} + \mathcal{O}(L_{CS}^2)\right)\right]\tilde{\psi}_{\lambda,\vec{k}} = 0$$

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PRD 110 (2024), 063512;
Universe 11 (2025), 15

For the graviton modes the momentum integrals are cutoff at the UV cutoff μ

The StRVM model is an EFT of a string theory → natural to identify the UV with the string scale

$$\mu \sim M_s$$

$$M_s \sim 10^{-1} M_{\rm Pl} < M_{\rm Pl}$$

Estimate the condensate within weak QG EFT

$$\langle R_{\rm CS}\rangle^I = -\frac{A}{\pi^2}\frac{\dot{b}_I}{M_{\rm Pl}}\left(\frac{H_I}{M_{\rm Pl}}\right)^3 \mu^4 < 0,$$

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Universe 11 (2025), 15

Include GW sources in the Condensate estimate

N. E. Mavromatos,
Lect. Notes Phys. 1017, 3 (2023).

Dorlis, Vlachos, NEM
PRD 110 (2024), 063512;
Universe 11 (2025), 15

$\mathcal{N}_I$ number of chiral GW sources @ onsert of RVM inflation (e.g. collapsing domain walls, primordial black holes, etc.)

$$\mathrm{Re}\,\langle R_{\rm CS}\rangle_I^{\rm total} = -\mathcal{N}_I\,\frac{A}{\pi^2}\,\frac{\dot{b}_I}{M_{\rm Pl}}\left(\frac{H_I}{M_{\rm Pl}}\right)^3\mu^4 < 0,$$

$\mathcal{N}_S$ number of chiral GW sources @ end of stiff-axion era → onsert of RVM inflation in StRVM

$$\mathrm{Re}\,\langle R_{\rm CS}\rangle_{\rm stiff}^{\rm total} = -\mathcal{N}_S\,\frac{30\sqrt{6}A\kappa^3\mu^4}{\pi^2}\,H_{\rm stiff}(\eta)^4,$$

Multiplicative factors containing number of sources can be understood in the framework of Path Integrals (PI) of weak gravitons, upon expanding about the mean field of the gCS condensate

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$$\mathcal{Z} = \int \mathcal{D}b\, \mathcal{D}g \;\; e^{iS[b,g]} = e^{iW\,[g,b]}$$

(cf. Part I adapted to CS gravity as EFT)

Expansion about a background metric (here FLRW), up to terms quadratic ((2)) in tensor fluctuations $h_{\mu\nu}$:

$$S[b, g+h] = S^{(0)}[b,g] + S^{(2)}[b,g,h] + \dots,$$

NB: Background fields are stationary points of PI exponent → satisfy classical eqs of motionI

$$-i\frac{\delta \ln \mathcal{Z}}{\delta b} = \frac{\delta W[g,b]}{\delta b} = 0\,, \qquad -i\frac{\delta \ln \mathcal{Z}}{\delta g_{\mu\nu}} = \frac{\delta W[g,b]}{\delta g_{\mu\nu}} = 0\,,$$

$$\mathcal{Z} = e^{iS^{(0)}[b,g]} \int \mathcal{D}h\ e^{iS^{(2)}[b,g,h]} = e^{iS^{(0)}[b,g]}\, \mathcal{Z}_h[b,g] \qquad \mathcal{Z}_h[b,g] = \int \mathcal{D}h\ e^{iS^{(2)}[b,g,h]}$$

$$S^{(2)}[b,g,h] \equiv S^{(2)}[b,g,h]\Big|_{A=0} - \int d^4x\, A\, b\, \mathcal{O}_h$$

$$\mathcal{O}_h = R_{\mu\nu\rho\sigma}\,{}^*R^{\mu\nu\rho\sigma}$$

expanded to O(h^2)
in tensor perturbations

* = **Hodge dual,** defined with the Minkowski Levi-Civita totally antisymmetric symbol

$${}^\star R_{\alpha\beta\gamma\delta} = \frac{1}{2} R_{\alpha\beta}{}^{\rho\sigma}\,\hat{\epsilon}_{\rho\sigma\gamma\delta}$$

Covariant Levi-Civita tensor

$$\varepsilon_{\mu\nu\alpha\beta} = \sqrt{-g(x)}\,\hat{\epsilon}_{\mu\nu\alpha\beta}$$

$$\boxed{\mathcal{Z} = e^{iS^{(0)}[b,g]} \int \mathcal{D}h\ e^{iS^{(2)}[b,g,h]\big|_{A=0} - iA\int d^4x\, b\, \mathcal{O}_h}}$$

$$0 = -i\frac{\delta \ln \mathcal{Z}}{\delta b} = \frac{\delta S^{(0)}[b,g]}{\delta b} + \frac{1}{\mathcal{Z}} \int \mathcal{D}h \left(\frac{\delta S^{(2)}[b,g,h]\big|_{A=0}}{\delta b} - A\,\mathcal{O}_h \right) e^{iS^{(2)}[b,g,h]\big|_{A=0} - iA\int d^4x\, \mathcal{O}_h\, b}$$

$$= \frac{\delta S^{(0)}[b,g]}{\delta b} + \left\langle \frac{\delta S^{(2)}[b,g,h]\big|_{A=0}}{\delta b} \right\rangle - A\langle \mathcal{O}_h \rangle$$

where

$$\boxed{\langle \mathcal{O}_h \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}h\ \mathcal{O}_h\ e^{iS^{(2)}[b,g,h]\big|_{A=0} - Ai\int d^4x\, \mathcal{O}_h\, b}}$$

Incorporation of number of sources $\mathcal{N} > 1$ in the PI:

Treat (chiral) GW from various sources as an **ideal gas** of **non-interacting tensor perturbations**

$$\mathcal{Z} = e^{iS^{(0)}[b,g]} \int \prod_{i=1}^{\mathcal{N}} \mathcal{D}h_i \; e^{i\sum_i S^{(2)}[b,g,h_i]\Big|_{A=0} - i\sum_i \int d^4x\, A\, \mathcal{O}_{h_i} b}$$

$$= e^{iS^{(0)}[b,g]} \left[\int \mathcal{D}h\, e^{iS^{(2)}[b,g,h]\Big|_{A=0} - Ai\int d^4x \mathcal{O}_h b} \right]^{\mathcal{N}} \qquad \mathcal{O}_h = R_{\mu\nu\rho\sigma}{}^{*}R^{\mu\nu\rho\sigma}$$

This explains, within the path integral formulation, the linear superposition of GW effects

$$\langle R_{\mu\nu\rho\sigma} \; {}^{*}R^{\mu\nu\rho\sigma} \rangle_n \equiv n \langle R_{\mu\nu\rho\sigma} \; {}^{*}R^{\mu\nu\rho\sigma} \rangle$$

proper density of GW sources

$$n = \mathcal{N}/\sqrt{-g}\,,$$

$$\int d^4x \mathcal{N}\, b(x) \langle R_{\mu\nu\rho\sigma} \, {}^{\star} R^{\mu\nu\rho\sigma} \rangle = \int d^4x \, \frac{1}{2} \mathcal{N}\, b(x)\, \langle R_{\mu\nu\rho\sigma} \hat{\epsilon}^{\mu\nu\alpha\beta}\, R_{\beta\alpha}{}^{\rho\sigma} \rangle = \int d^4x \sqrt{-g}\, \mathcal{N}\, b(x)\, R_{\mu\nu\rho\sigma}\, \widetilde{R}^{\mu\nu\rho\sigma}$$

Estimates of chiral GW Sources @ beginning of RVM inflation from smooth passage from (pre-inflationary) stiff era to RVCM inflation?

Dorlis, NEM, Vlachos PRD110 (2024); Universe 11 (2025) 15

@ transition era stiff-axion → RVM Inflation

$$\text{Re}\,\langle R_{\rm CS}\rangle_I^{\rm total} = -\mathcal{N}_I \frac{A}{\pi^2}\frac{\dot{b}_I}{M_{\rm Pl}}\left(\frac{H_I}{M_{\rm Pl}}\right)^3 \mu^4 < 0,$$

$$= \text{Re}\,\langle R_{\rm CS}\rangle_{\rm stiff}^{\rm total} = -\mathcal{N}_S \frac{30\sqrt{6}A\kappa^3\mu^4}{\pi^2} H_{\rm stiff}(\eta)^4,$$

Estimates of chiral GW Sources @ beginning of RVM inflation from smooth passage from (pre-inflationary) stiff era to RVCM inflation?

Dorlis, NEM, Vlachos PRD110 (2024); Universe 11 (2025) 15

$O(H_I^4)$

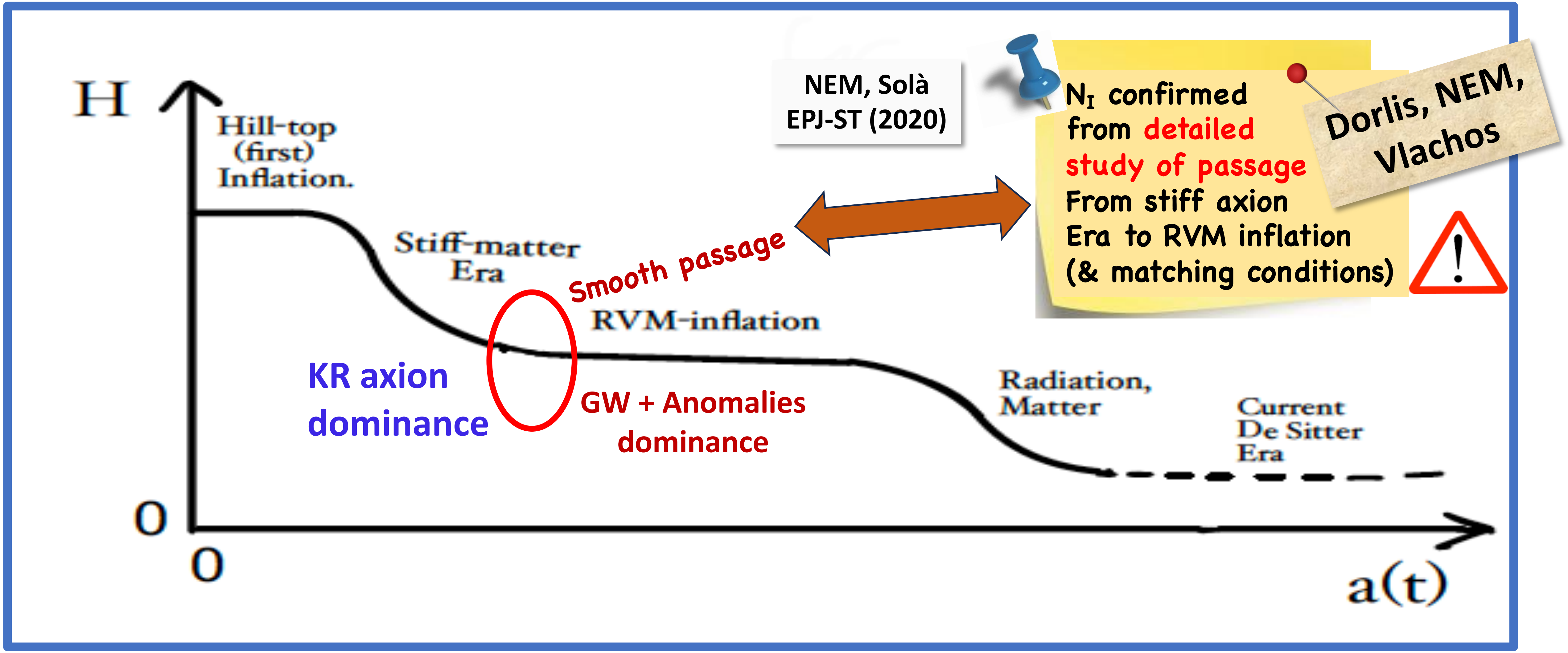
@ transition era stiff-axion → RVM Inflation

$$\mathrm{Re}\,\langle R_{\rm CS}\rangle_I^{\rm total} = -\mathcal{N}_I \frac{A}{\pi^2} \frac{\dot{b}_I}{M_{\rm Pl}} \left(\frac{H_I}{M_{\rm Pl}}\right)^3 \mu^4 < 0,$$

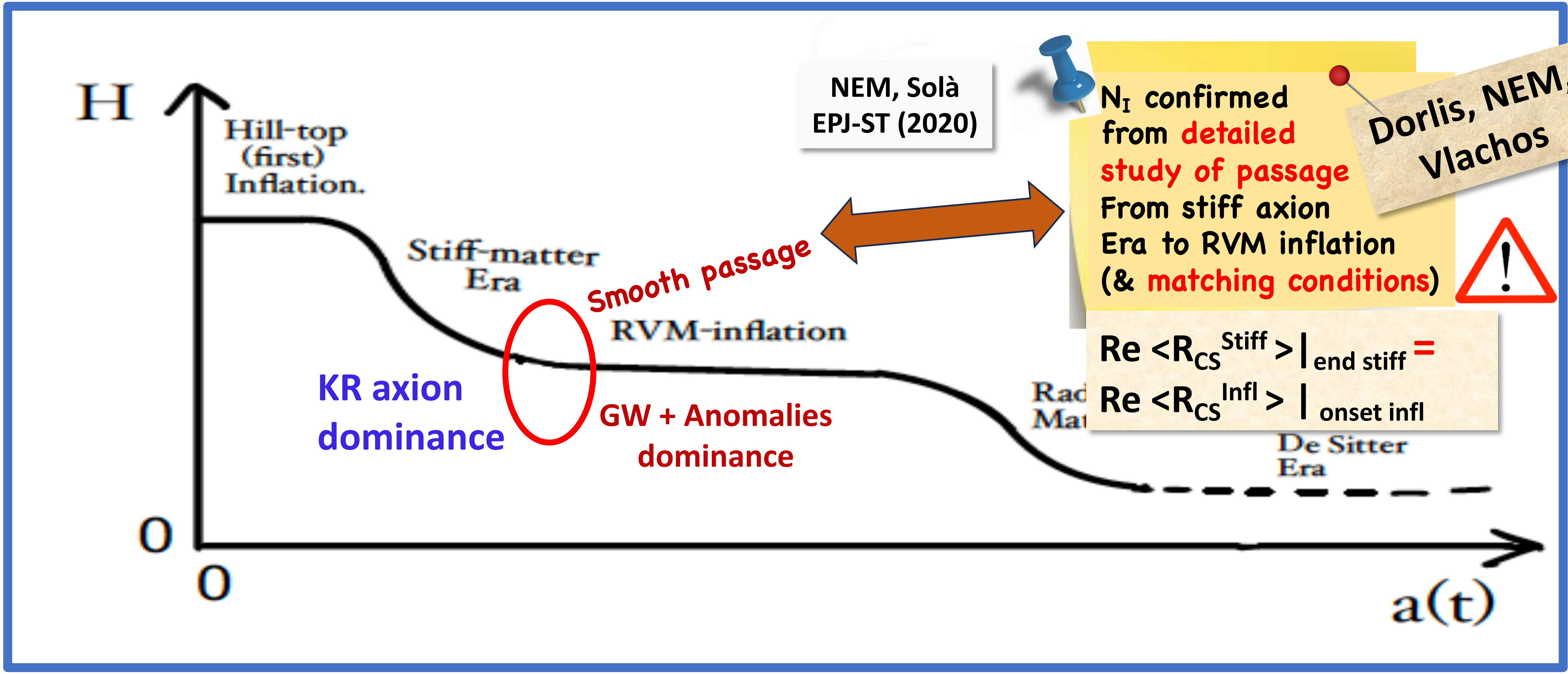
$$= \mathrm{Re}\,\langle R_{\rm CS}\rangle_{\rm stiff}^{\rm total} = -\mathcal{N}_S \frac{30\sqrt{6} A \kappa^3 \mu^4}{\pi^2} H_{\rm stiff}(\eta)^4,$$

Dynamical system analysis → $\dot{b}_I \sim 10^{-1} H_I M_{\rm Pl}$, also Inflation cf. spares

$\boxed{N_I \gtrsim O(10^{16})}$ @ the beginning of RVM inflation if O(1)



$$\boxed{N_I \gtrsim O(10^{16})}$$ @ the beginning of RVM inflation



String KR axions, Condensates & Inflation

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}\,R + \frac{1}{2}\,\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\,\frac{\alpha'}{96\,\kappa}\,b(x)\,\Big(R_{\mu\nu\rho\sigma}\,\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\,\widetilde{F}^{\mu\nu}\Big) + \dots\Big]$$

Primordial string Universe Gravitational Waves (GW): e.g. from collapse of (rotating) primordial black holes (PBH) sourced by Torsion-induced axion field *b(x)* can induce **condensates** of gravitational Chern-Simons terms

Condensates lead to **linear axion** potentials

$$V(b) \ni b(x)\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle$$

We computed using weak (perturbative) Quantum gravity techniques With chiral GW perturbation modes

$\mathbf{O(H_I{}^4)}$

Lyth, Rodriguez, Quimbay
Alexander, Peskin, Sheikh-Jabbari
Dorlis, Vlachos, NEM

String KR axions, Condensates & Inflation

$$S_B^{\rm eff} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2\kappa^2}R + \frac{1}{2}\partial_\mu b\,\partial^\mu b + \sqrt{\frac{2}{3}}\frac{\alpha'}{96\,\kappa}\,b(x)\left(R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma} - F_{\mu\nu}\widetilde{F}^{\mu\nu}\right) + \dots\Big]$$

Primordial string Universe Gravitational … e.g. from collapse of (rotating) primordial black holes (PBH)

sourced by Torsion-induced ax… **condensates** of gravitational Chern-Simons terms

Resulting Vacuum Energy of RVM type (Solà …)

$\rho = c_1 H^2 + c_3 H^4$

Condensates … **…tentials**

$$V(b) \ni \dots_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle$$

We computed using weak (perturbative) Quantum gravity techniques With chiral GW perturbation modes

$O(H_I{}^4)$

Inflation → cf. dynamical system analysis → finite duration (spares)

Lyth, Rodriguez, Quimbay
Alexander, Peskin, Sheikh-Jabbari
Dorlis, Vlachos, NEM

CONDENSATE WITH CUTOFF GRAVITON MODES HAS **IMAGINARY PARTS**

(**ENVIRONMENT** OF MODES WITH MOMENTA ABOVE THE CUTOFF $\boldsymbol{\mu}$)

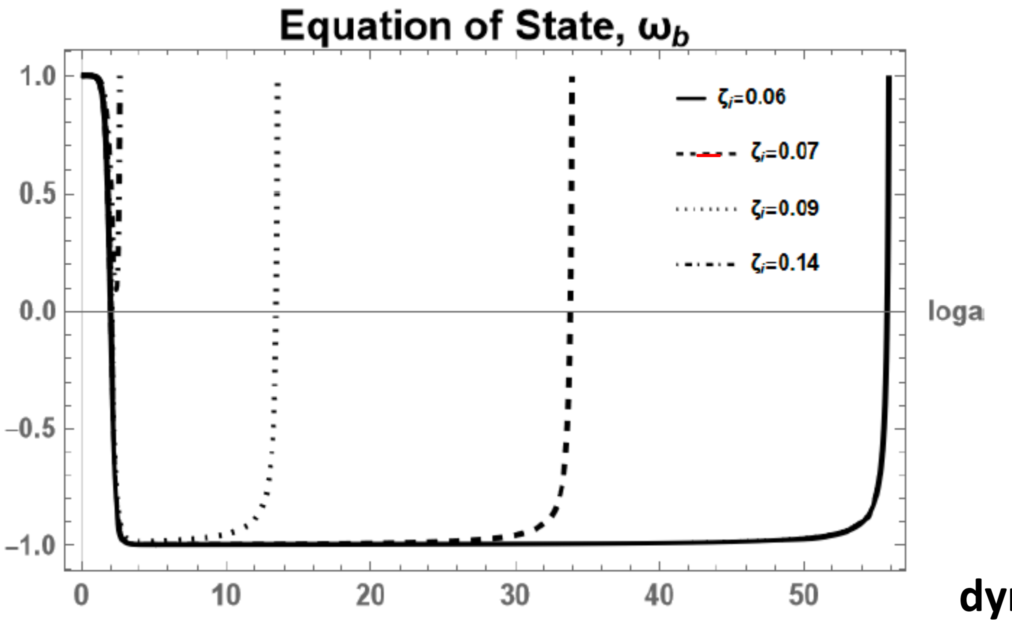
$M_s = O(10^{-1})\ M_{Pl}$

Consistent with swampland

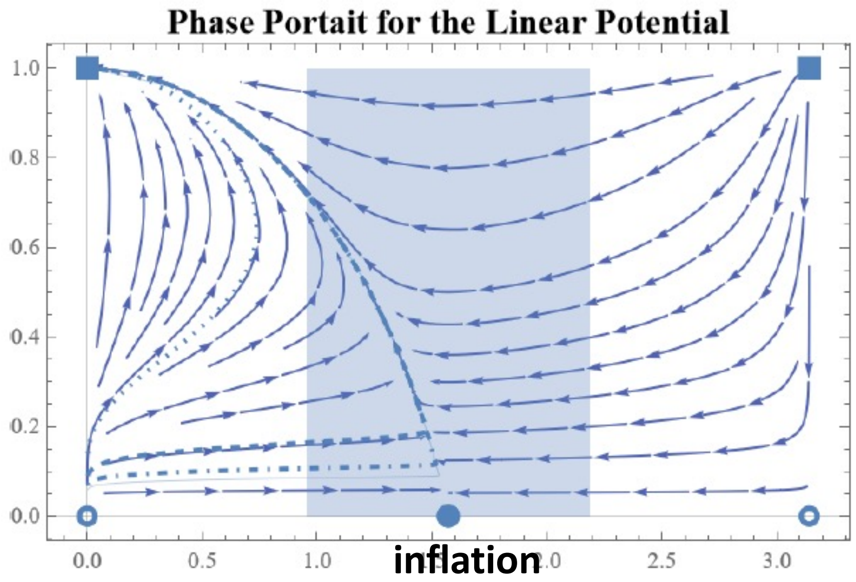
INSTABILITY OF CONDENSATE PHASE → **FINITE LIFE TIME OF THIS PHASE**

Dorlis, Vlachos, NEM Universe 11 (2025), 15 2411.12519 [gr-qc]

→ CONSISTENT WITH ***50 e-FOLDINGS*** AS STEMS FROM DYNAMICAL SYSTEM ANALYSIS !



$\zeta - \varphi$ plane.



dynamical systems

Vafa, Ooguri *et al.*
Quantum Gravity Inconsistent with ***eternal*** *de Sitter vacua*

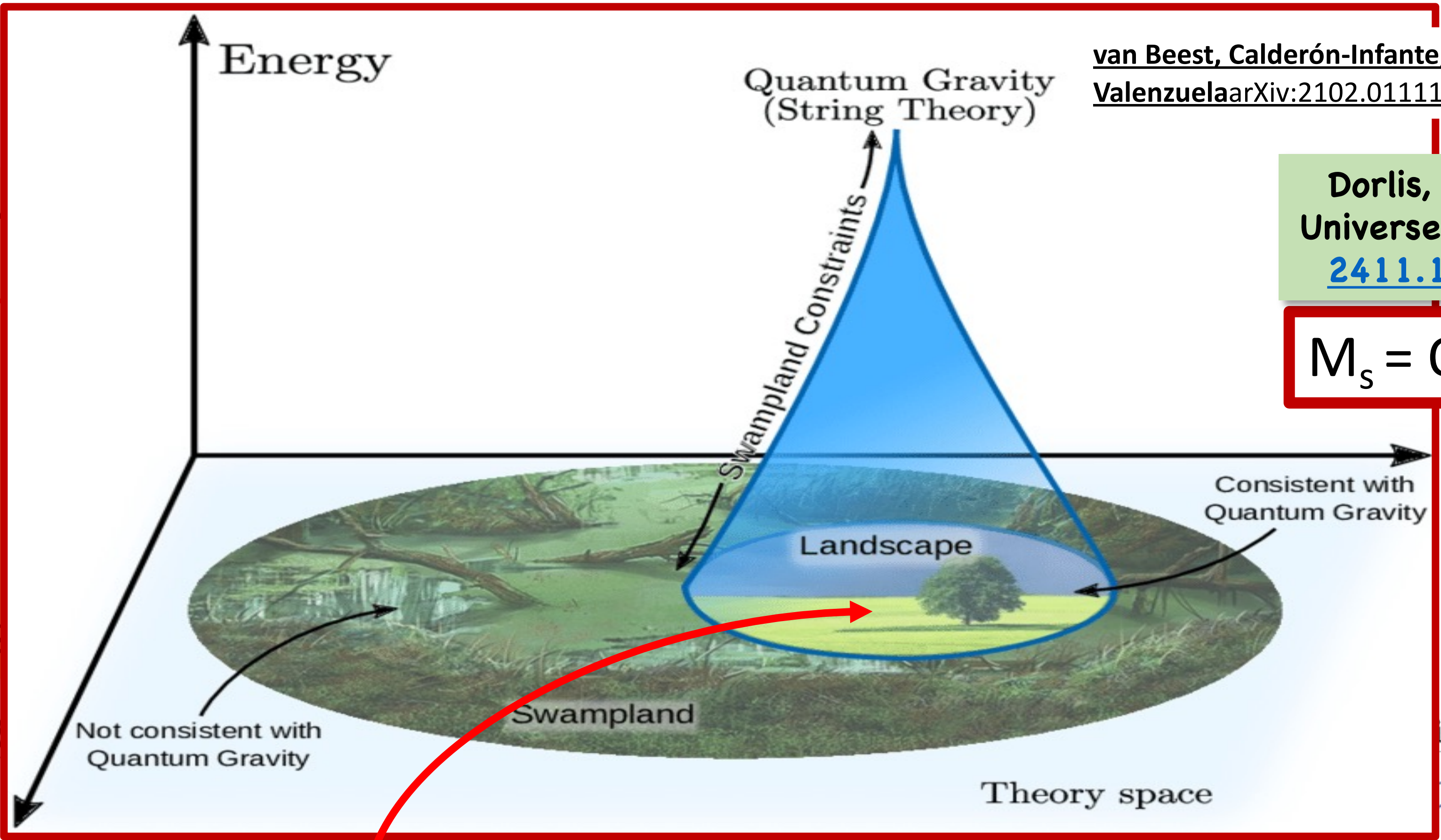
Consistent with swampland

...DENSATE W...

(ENVIRONMENT...

INSTABILITY OF

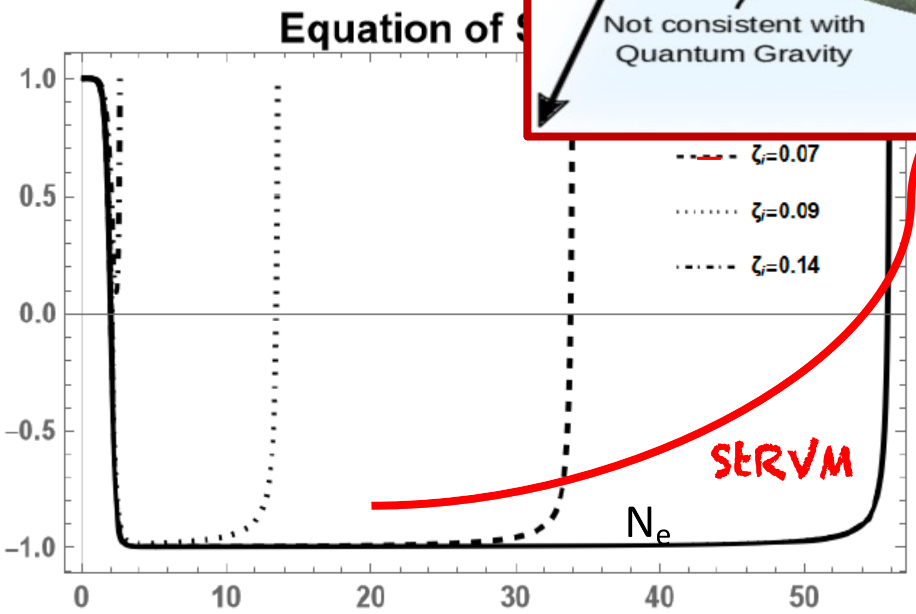
→ CONSISTENT ... SYSTEM ANALYS...



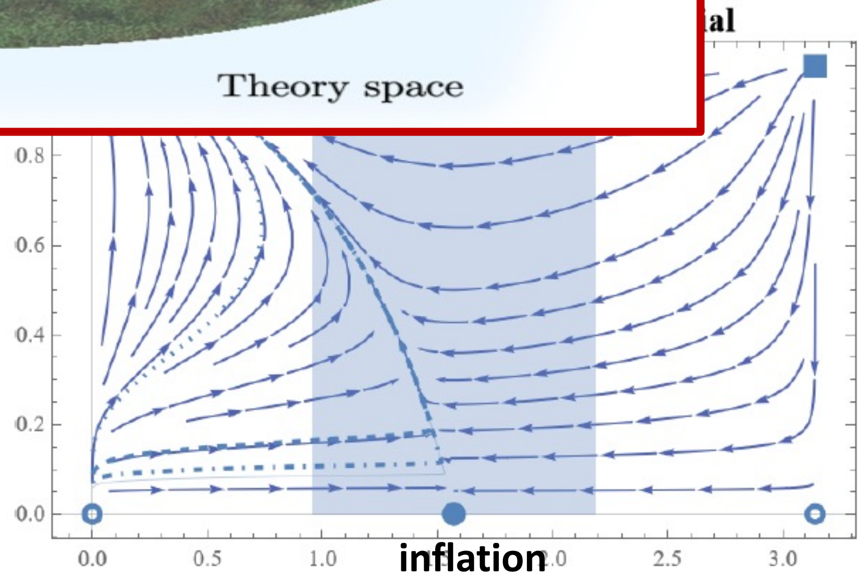
van Beest, Calderón-Infante, Mirfendereski, Valenzuela arXiv:2102.01111 [hep-th]

Dorlis, Vlachos, NEM Universe 11 (2025), 15 2411.12519 [gr-qc]

$M_s = O(10^{-1})\, M_{Pl}$



loga



inflation

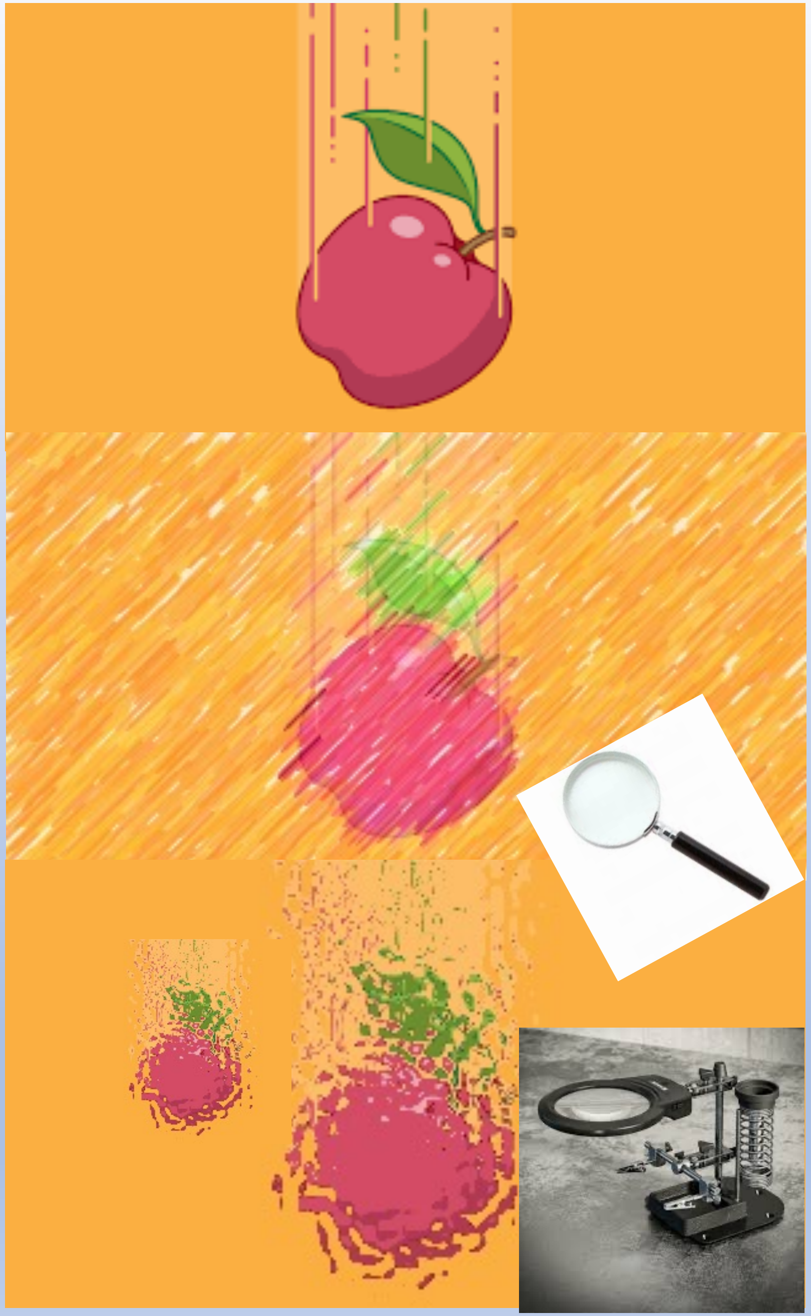
dynamical systems

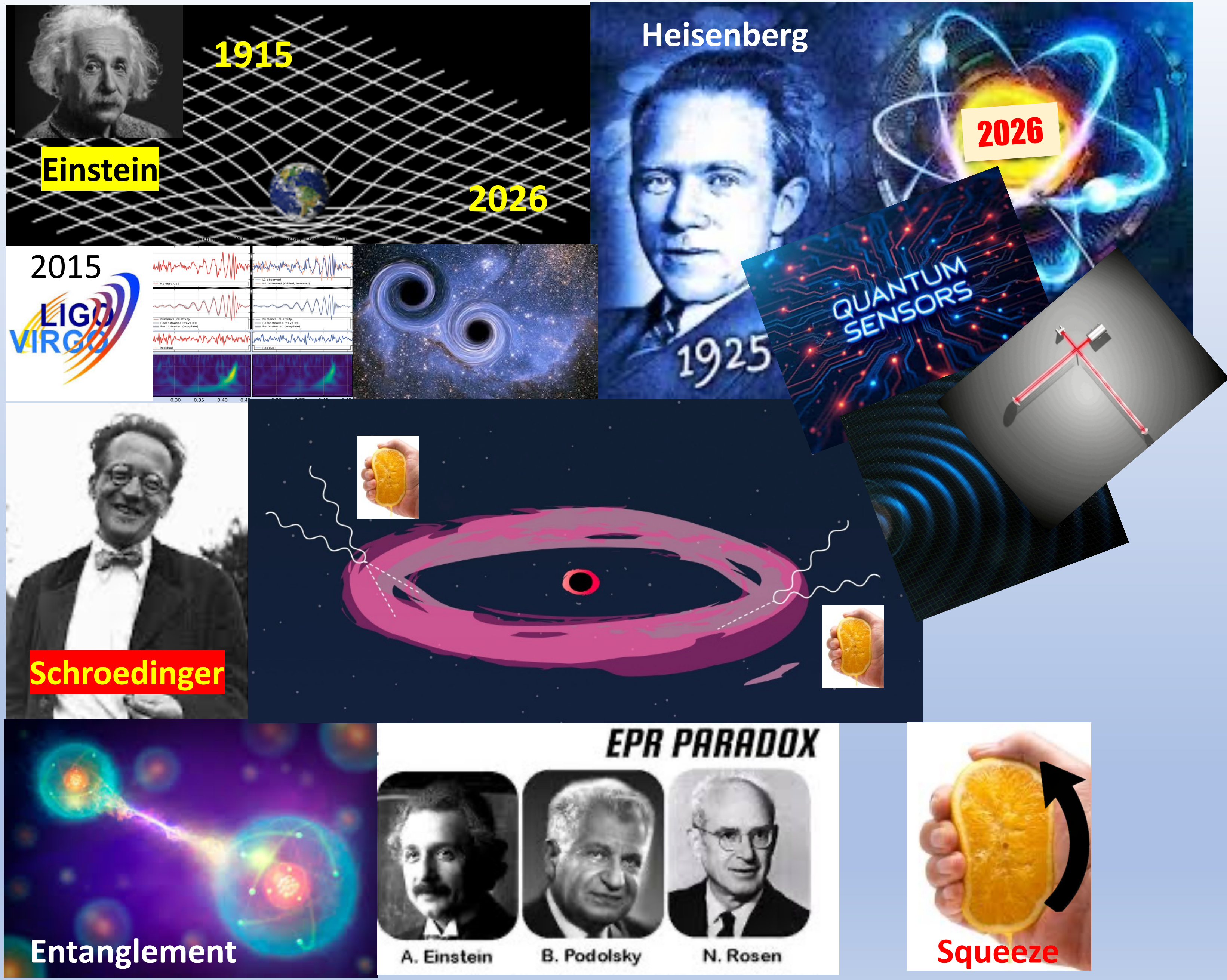
Other applications of weak Gravity as Effective Field Theory

$\boxed{\text{(ii) Axions clouds in Rotating Black Holes}}$

→ Superradiance Phenomenon

→ $\boxed{\text{production of Squeezed entangled gravitons}}$ in gravitational waves




Einstein
1915
2026
Heisenberg
2026
1925
QUANTUM SENSORS
2015
LIGO
VIRGO
Schroedinger
Entanglement
EPR PARADOX
A. Einstein
B. Podolsky
N. Rosen
Squeeze

Usual assumption in the structure of GW is that they consist of coherent states of gravitons, which behave almost classically

But there is always a small admixture of squeezed graviton states

The latter can - under some circumstances - lead to observable phenomena due to specific enhancement factors - specifically if they are produced (in entangled polarization configurations) from superradiant axion clouds of astrophysical rotating black holes.

Outline of remainder of Lecture 2

- Squeezing in Quantum Optics/Quantum Information theory
- The Gravitational set up: why we need squeezing for QG tests? where? Rotating (Kerr-type) Black Holes + Massive-Axion superradiant clouds (collective phenomena enhancement of QG effects)
- Massive Axions → production of squeezed (polarization) entangled multi-mode quantum graviton states
  Context: weak quantum gravity effective field theory
  Analogies with Quantum Optics
- The figure of merit (Number N_{sg} of Squeezed Gravitons produced)

  How much can a graviton be squeezed? Argue that N_{sg} can be much larger than 1
- Observational Prospects – non-observation of squeezing today → Current Exclusion limits on longevity of Axionic Clouds

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Squeezed Quantum Coherent State:

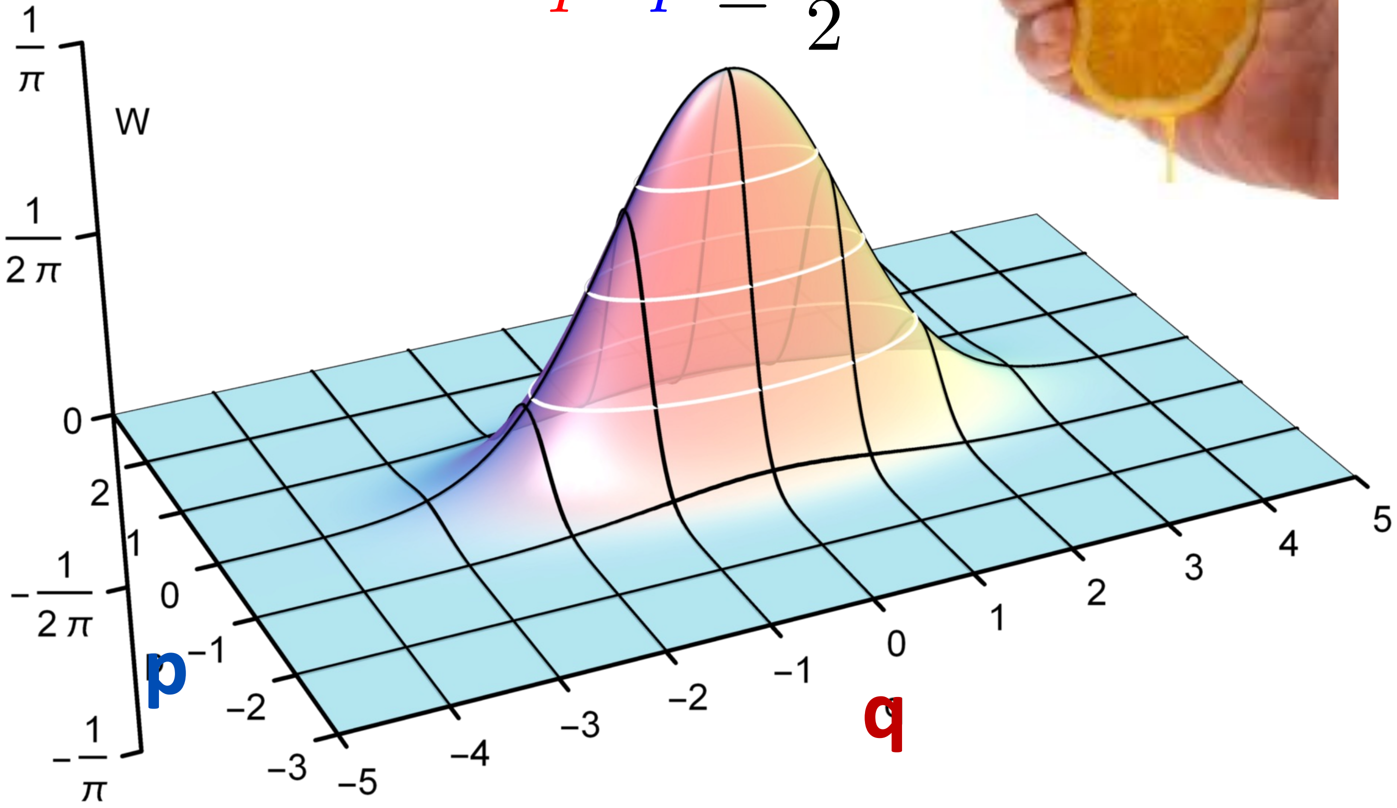
A quantum state usually described by **two non-commuting Observables,** which has **continuous spectra of eigenvalues and:**

a **standard deviation below that of the ground state** for **one of the operators or** for a **linear combination of the two**, *i.e.* the **circle** denoting the **uncertainty** of a **coherent state** in the **quadrature phase space** has been "**squeezed**" to an **ellipse of the same area**.

A **squeezed state** does **not necessarily saturate** the uncertainty principle.

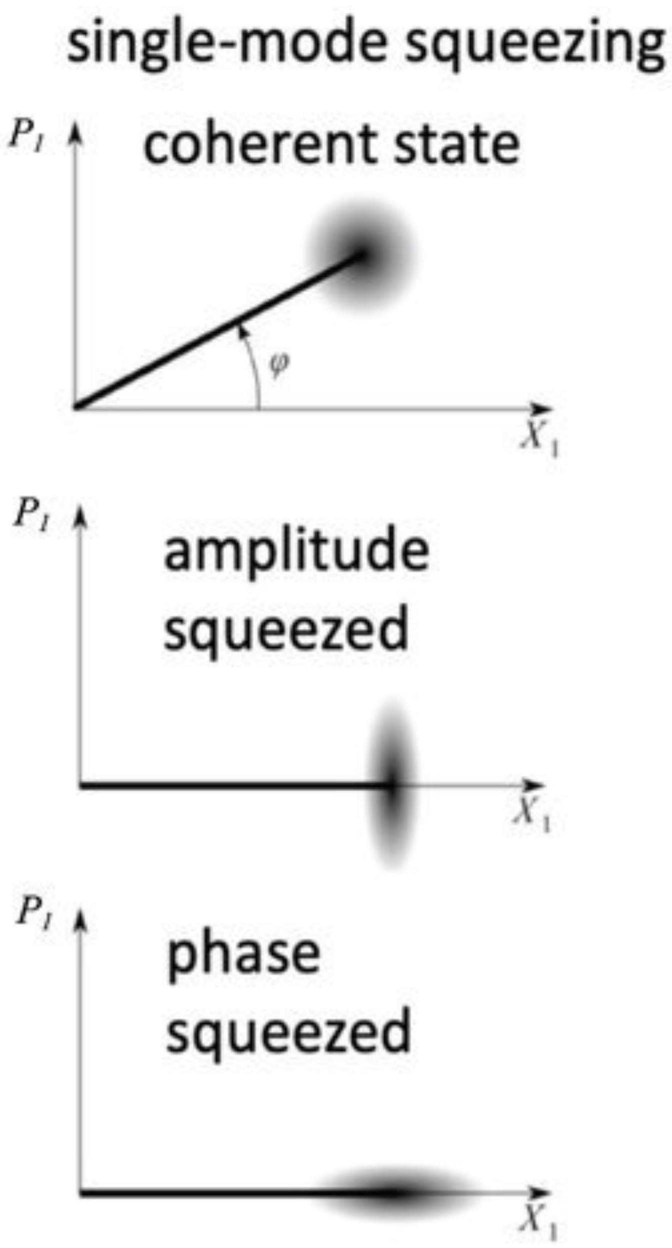
$$\left[\hat{q},\,\hat{p}\right] = i\hbar$$

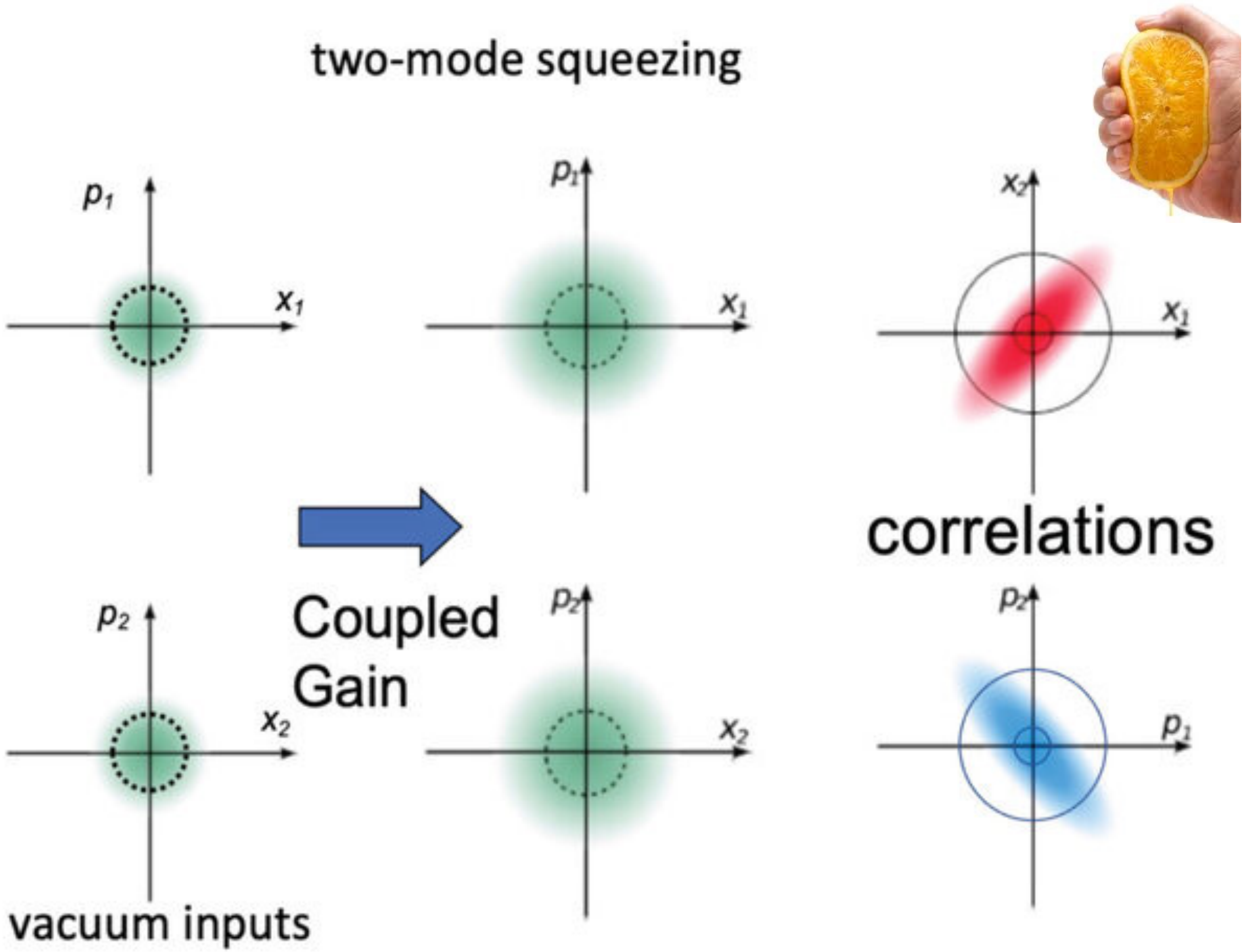
$$\Delta q\,\Delta p \geq \frac{\hbar}{2}$$



Wigner phase space distribution of a squeezed state of light with ζ=0.5

e.g.





Glorieux, Q.., Aladjidi, T. , Lett, P. & Kaiser, R.,
New Journal of Physics 25 (2023) 10.1088/1367-2630/acce5a

Squeezed states of Harmonic Oscillator

wavefunction

similar to coherent states, but product of variances does not saturate the uncertainty principle and it depends on time

$$\psi_s(x,0) \propto \exp\left[\frac{i}{\hbar}p_0(0)x - \frac{\beta}{2\hbar}m\omega_0(x - x_0(0))^2\right]$$

β is the "squeezing parameter".

Solution of time dependent Schroedinger 's equation yields:

$$\psi_s(x,t) \propto \exp\left[i\,\epsilon(t) + \frac{i}{\hbar}p_0(t)x - \frac{S(t)}{2\hbar}m\omega_0(x - x_0(t))^2\right]$$

$$\epsilon(t) = -\frac{1}{2}\omega_0 \int_0^t dt'\,S(t') - \frac{1}{2\hbar}x_0(t)p_0(t)$$

Squeezing function

$$S(t) = \mathrm{Tanh}\left(\mathrm{Tanh}^{-1}(\beta) + i\,\omega_0\,t\right)$$

$$S(0) = \beta\,.$$

Squeezed states of Harmonic Oscillator

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β is the "squeezing parameter". $\quad \beta \neq 1$

$\beta = 1 \Rightarrow$ retrieve the coherent state

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Formal definitions & Properties

Ground State

$$\sigma_x \sigma_p = \frac{\hbar}{2}$$

minimum uncertainties

Canonical Coherent state:

$$\sigma_x \sigma_p = \frac{\hbar}{2}$$

closest to a classical state
minimum uncertainties

Squeezed States

e.g. harmonic oscillator β : squeezing parameter

$$\sigma_x \sigma_p = \frac{\hbar}{2}\sqrt{\frac{1+6\beta^2+\beta^4-(\beta^2-1)^2\cos(4\omega_0 t)}{8\beta^2}}.$$

$$(\sigma_x \sigma_p)_{\text{min}} = \frac{\hbar}{2}, \qquad (\sigma_x \sigma_p)_{\text{max}} = \frac{\hbar}{4}\left(\beta^{-1}+\beta\right)$$

time-dependent product of variances,
do not saturate the uncertainty principle

β → 1 → coherent state of harmonic oscillator

Hamiltonian

In terms of phase-space quadrature operators

$$\mathcal{H} = \frac{1}{2}\sum_{i,j}\left[\mathcal{G}_{ij}\hat{a}_i^\dagger \hat{a}_j^\dagger + \mathcal{G}_{ij}^*\hat{a}_i\hat{a}_j\right].$$

$$a_j = \frac{q_j + ip_j}{\sqrt{2}}, \; a_j^\dagger = \frac{q_j - ip_j}{\sqrt{2}}$$

Formal definitions & Properties

Squeezed States

Squeezing Operator
e.g. single-mode (j=1)

$$\boxed{S(\zeta) = e^{\frac{1}{2}(\zeta^{\star}\,(\hat{a})^2 - \zeta(\hat{a}^{\dagger})^2)}\,, \qquad \zeta = r\,e^{i\,\theta}}$$

$$S^{\dagger}(\zeta) = S^{-1}(\zeta)\,, \qquad S^{\dagger}\,\hat{a}\,S(\zeta) = \hat{a}\cosh(r) - \hat{a}^{\dagger}\,e^{-i\,\theta}\sinh(r)$$

Mean "photon" number

Bogolubov trnsf.

$$\boxed{\langle N\rangle = \langle\zeta|\hat{a}^{\dagger}\,\hat{a}|\zeta\rangle = \sinh^2(r)\,,} \qquad |\zeta\rangle = S(\zeta)|0\rangle$$

$$|\zeta\rangle = \frac{1}{\sqrt{\cosh r}}\sum_{n=0}^{\infty}\frac{\sqrt{(2n)!}}{2^n n!}(-e^{i\theta}\tanh r)^n|2n\rangle$$

e.g. harmonic oscillator β : squeezing parameter

$$\boxed{\sigma_x\,\sigma_p = \frac{\hbar}{2}\sqrt{\frac{1+6\beta^2+\beta^4-(\beta^2-1)^2\cos(4\omega_0 t)}{8\beta^2}}.}$$

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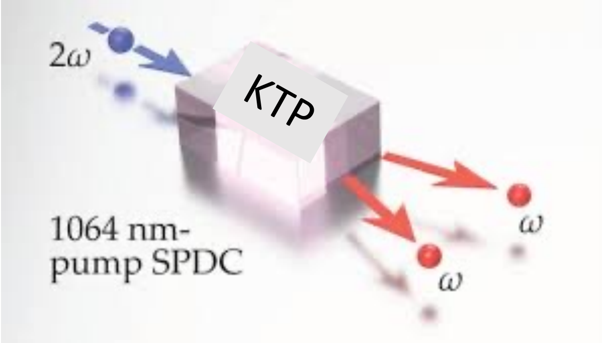
Generation of Squeezed state of light...

(i) Spontaneous Parametric Down Conversion (SPDC)

Process: A **high-energy pump photon** interacts with a **nonlinear crystal** (such as KTP - **Potassium titanyl phosphate** ($KTiOPO_4$))



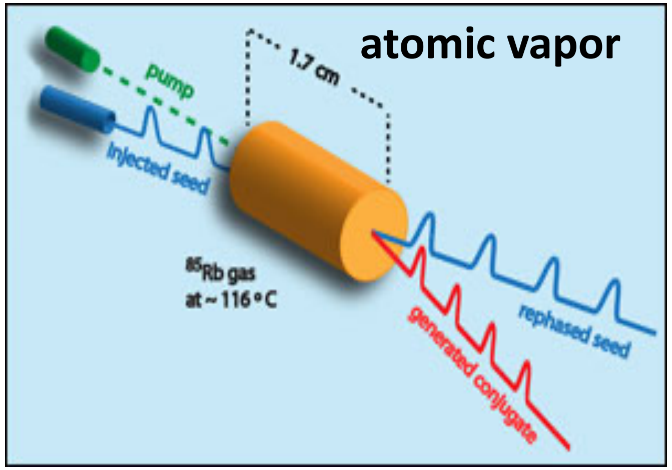
and **splits spontaneously** into a **pair of lower-energy photons**, the **signal** and **idler.**

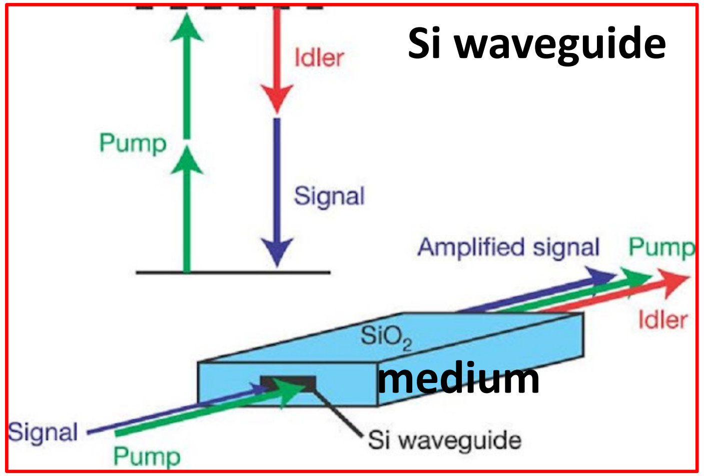
Creation of Squeezing: The so-created **photon pairs** are **quantum correlated** → such **quantum correlations** manifest themselves as **squeezing** in the **electric field** quadratures.

NB: If an **optical parametric oscillator** is operated **below** its oscillation **threshold** → can generate **squeezed vacuum** with significant noise reduction.
If operated **above** the **threshold** → can produce **bright squeezed light**.

Generation of Squeezed state of light…

(ii) Four Wave Mixing (FWM)





Process: **two pump photons** at frequency ω_p interact in **a nonlinear medium** to **generate a pair of signal and idler photons**.

What media? optical fibers or **atomic vapors** (*e.g.*, rubidium or sodium), or **silicon waveguides**

Creation of Squeezing: FWM in a **cavity** can produce **squeezed light**.

Coherent State Input: seeding the FWM process with **a coherent beam** → **bright squeezed coherent** state generation

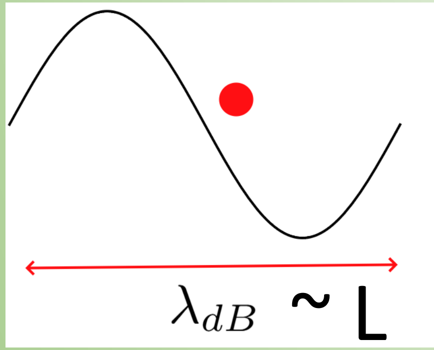
Squeezing of Gravitons
in
(weak) Quantum Gravity EFT:
Why ?
and
the ingredients

First things first: **why** look for **squeezed entangled** states of **gravitons**?

First things first: **why** look for **squeezed entangled** states of **gravitons**?

Single-mode graviton-state searches may be too difficult to yield observational results in the foreseeable future

F. Dyson, IJ.MP A 28, 1330041 (2013)



Single quantum regime

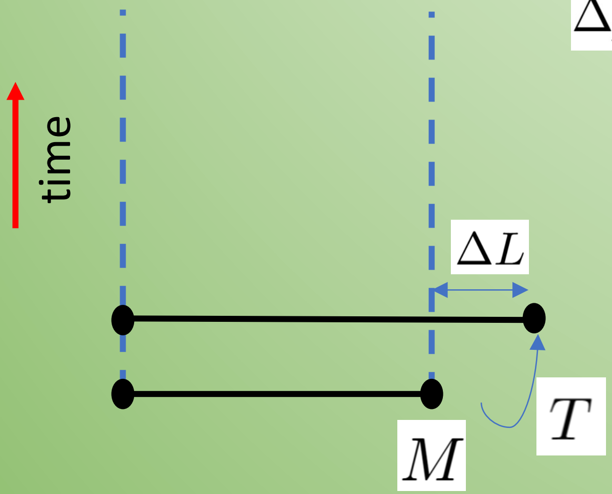
Measurement of single graviton by a detector:

Absorption cross section σ ~ $(L_{planck})^2$

Hugely Massive detectors required

Planck length accuracy

$$\Delta L \sim M_{Pl}^{-1} = L_{Pl}$$



$$\Delta p \Delta x \geq 1/2$$

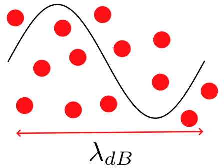
$$M \frac{\Delta L}{T} \Delta L \geq \frac{1}{2}$$

$$T \geq L$$

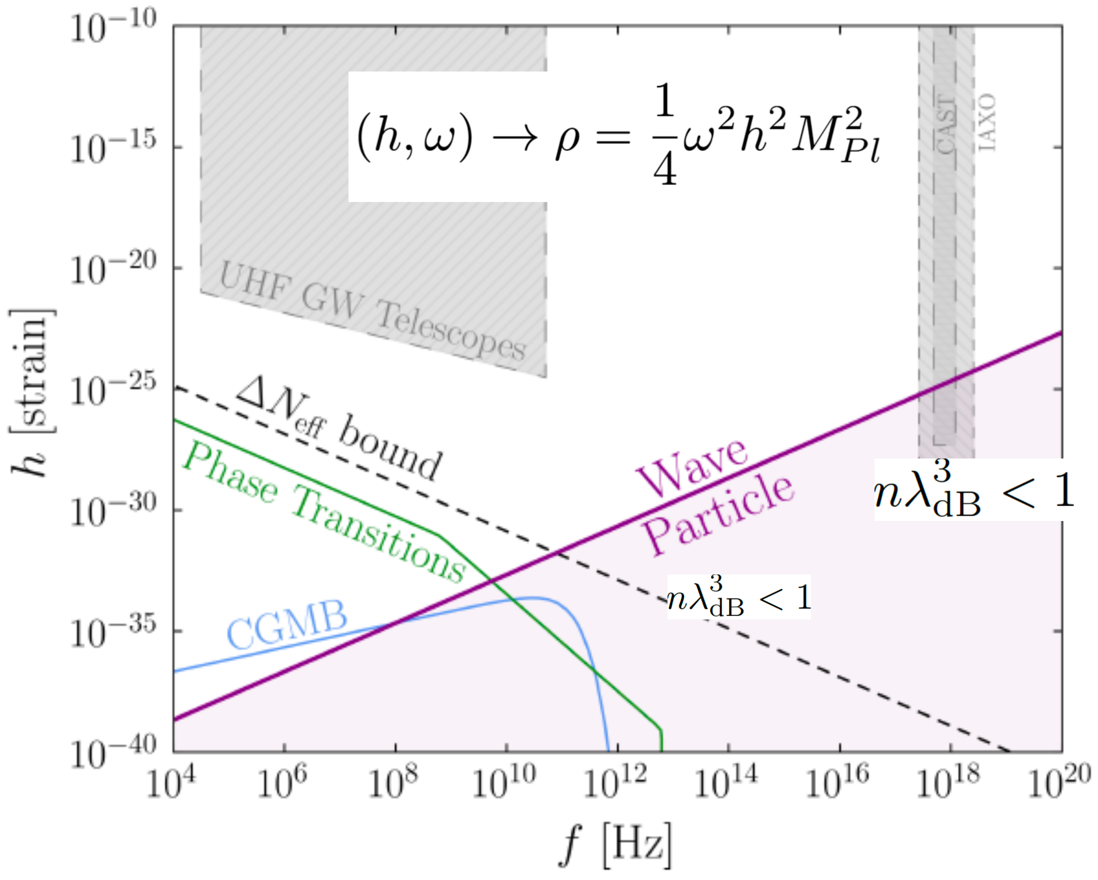
$$L \leq 2MG$$

Collapse of the apparatus into a black hole !

NB: **LIGO detected GWs contain a macroscopically large number of gravitons!**



$$n\lambda_{\rm dB}^3 = \frac{\pi h^2 M_{\rm Pl}^2}{2f^2} \simeq 2\times 10^{35}\left(\frac{h}{10^{-22}}\right)^2\left(\frac{1\ {\rm kHz}}{f}\right)^2$$



Signal duration comparable to or longer than the measurement time (for CAST this is about a year)

@ much higher frequencies the number density of gravitons in a GW is significantly diluted

Sensitivity of CERN-AXION-SOLAR-TELESCOPE (CAST)

@ a magnetic field of order 10 Tesla & X-ray photodetector ($f \sim 10^{18}$ Hz) → relevant interactions between gravitons and electromagnetic radiation

$$\mathcal{L} = \frac{1}{2} h_{\mu\nu} T^{\mu\nu}_{\rm em}$$

→ sensitivity to strains $h \sim 10^{-27}$ → $n\lambda_{\rm dB}^3 < 1$!!!
(highly dilute distribution of gravitons)

→ & also of its successor IAXO

→ so in principle CAST/IAXO can **detect individual gravitons** but there are **no known sources of gravitational radiation @ such frequencies**

But classical GW can also mimic such a detector response

D. Carney, V. Domcke, N. L. Rodd, PRD 109, 044009 (2024)

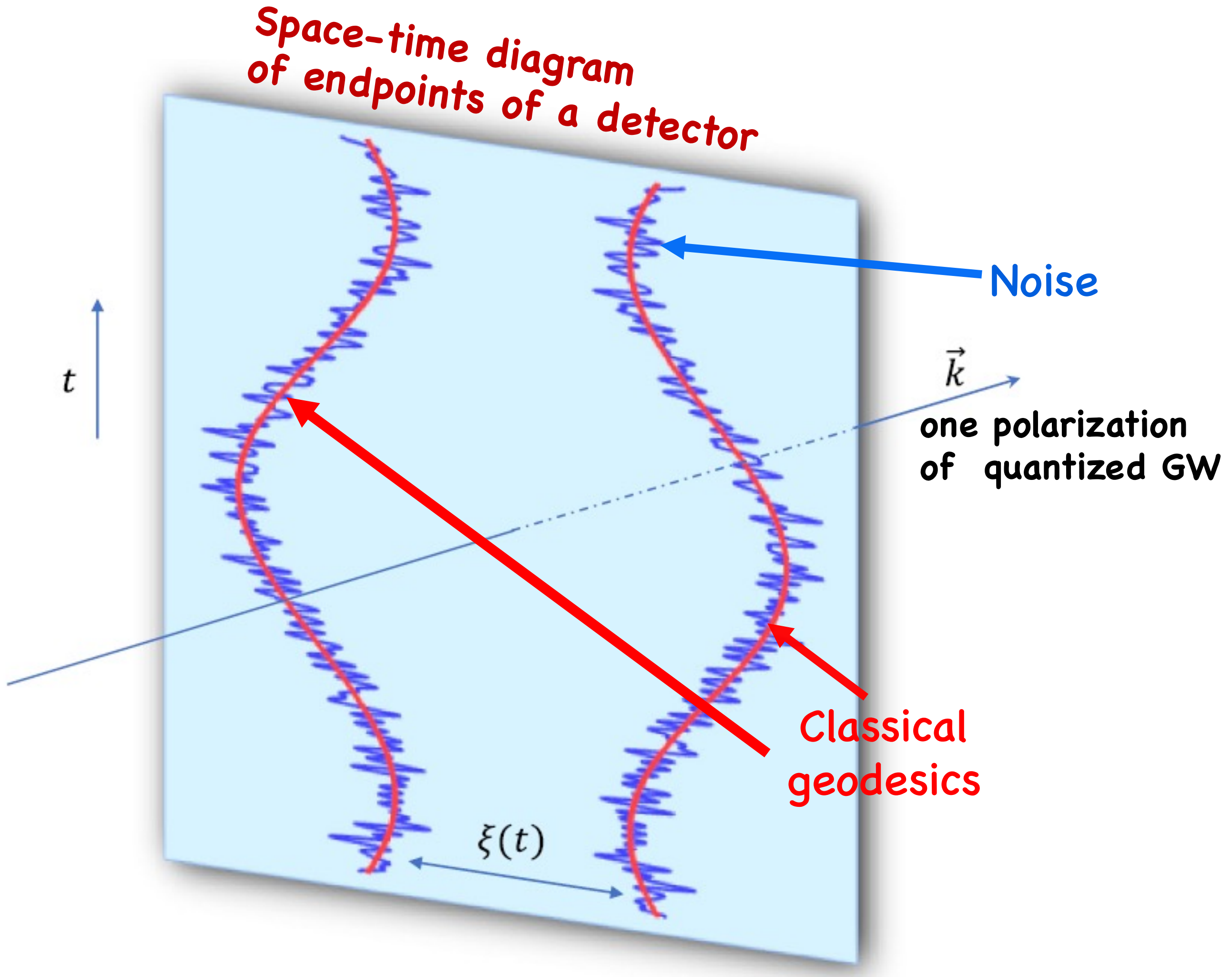
QG as ``*noise*'' in interferometers.

Enhanced signal for **squeezed single-mode gravitons**

Amelino-Camelia, Nature 398, 216 (1999); PRD 62, 024015 (2000

Parikh, Wilczek, and Zahariade, PRL 127, 081602 (2021); PRD 104, 046021 (2021)

Parikh, Wilczek, Zahariade
PRL 127, 081602 (2021)
PRD 104, 046021 (2021)

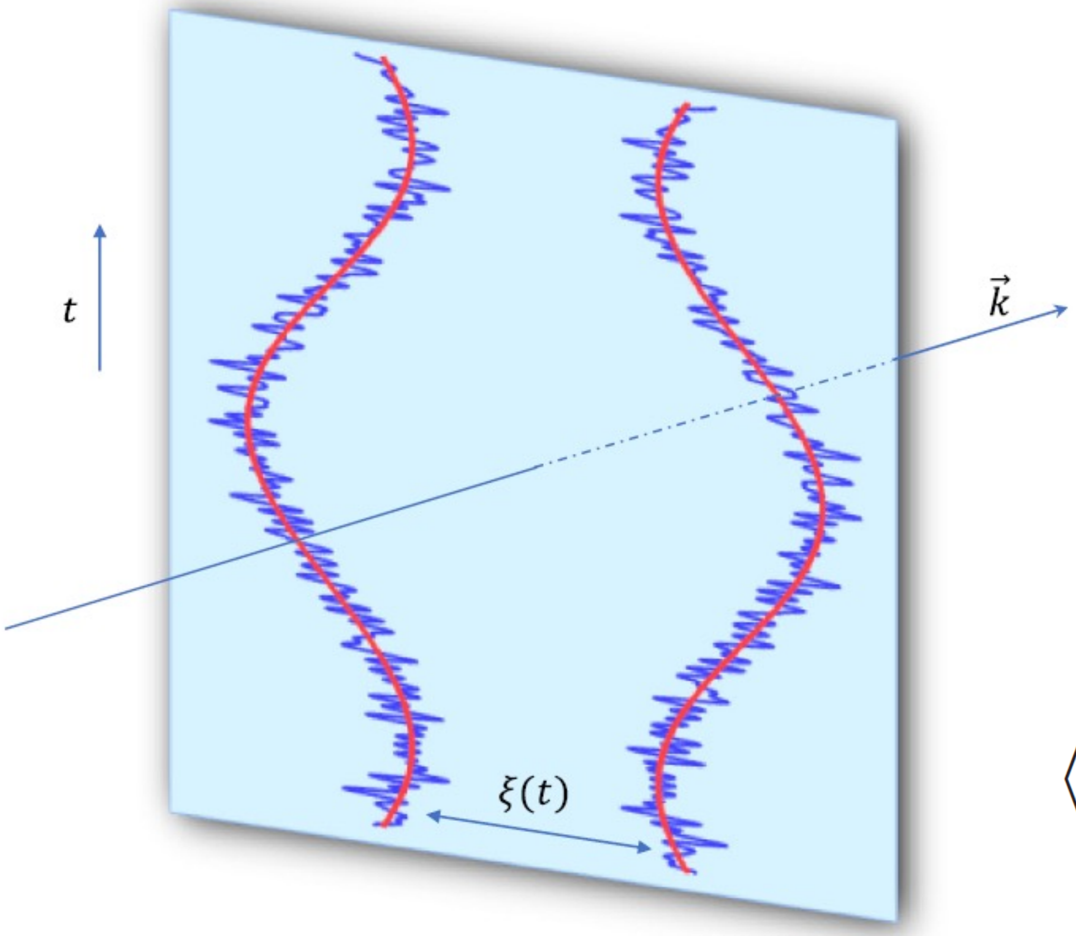


Falling particles subject to ``Noise"
due to quantum fluctuations of spacetime ;
Noise characteristics depend on graviton state

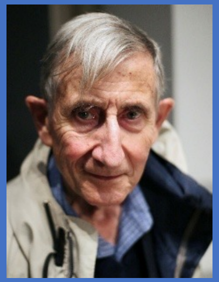
Idealization:

2 mirrors at the **endpoints** of
the **arm of an interferometer**
are idealized as **falling particles**
in **weak gravitational field**

Parikh, Wilczek, Zahariade
PRL 127, 081602 (2021)
PRD 104, 046021 (2021)



Vacuum: $\sigma_0 \sim L_p \, \xi_0 \, \omega_{max} \sim L_p$



No !

Squeezing: $\sigma_{\text{squeezed}} = \sigma_0 \sqrt{\cosh 2r}$

Perhaps?

r = squeezing parameter

Exponential enhancement !
potential detection if r >> 1

GW

$$\langle \xi \rangle = (1 + h(t)/2)\xi_0$$ classical

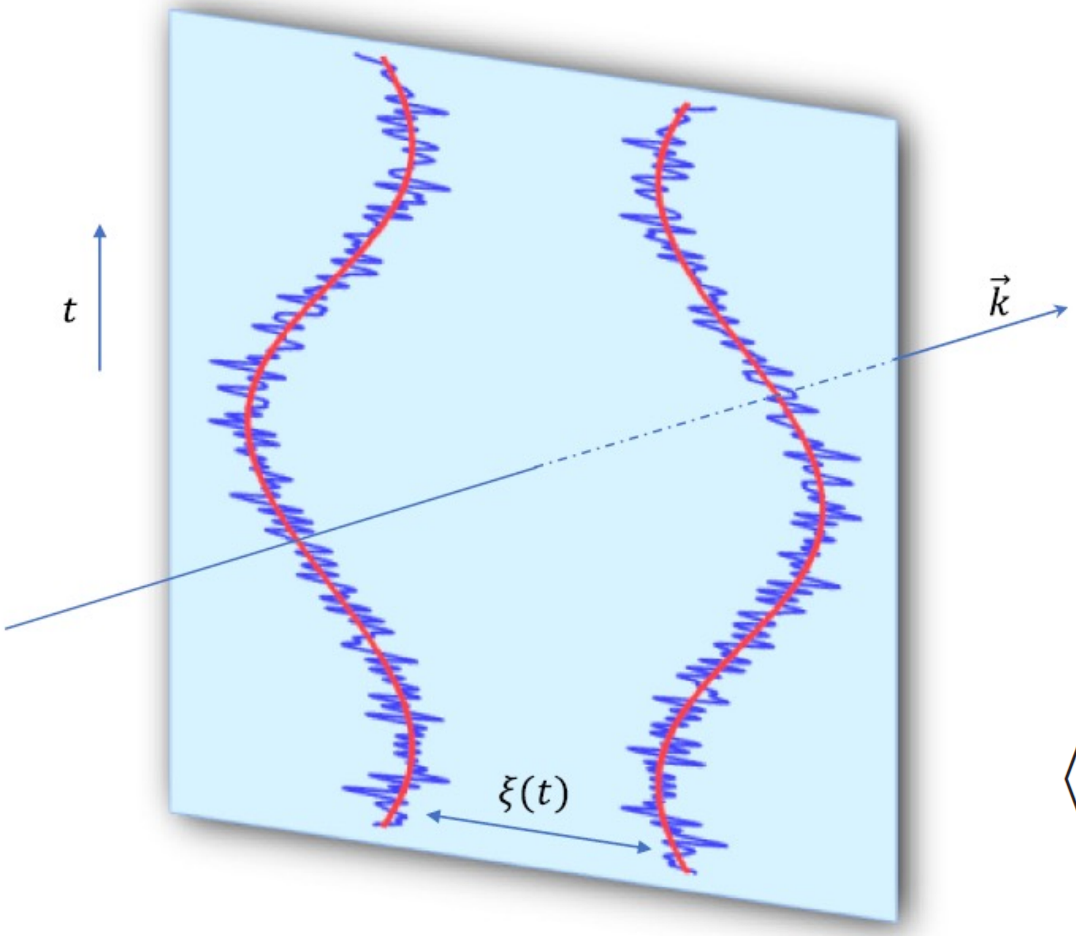
$$\sigma^2 = \xi_0^2 \langle N^2 \rangle / 4$$

Non-zero variance.
Purely quantum effect!

stochastic noise

courtesy P. Dorlis

Parikh, Wilczek, Zahariade
PRL 127, 081602 (2021)
PRD 104, 046021 (2021)



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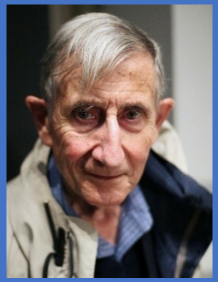

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Perhaps?

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potential detection if r >> 1

How much can a graviton be squeezed?

$$\langle \xi \rangle = (1 + h(t)/2)\xi_0$$ classi

$$\sigma^2 = \xi_0^2 \langle N^2 \rangle / 4$$ Non-zer riance. Purel antum effect!

sources of such large squeezing?

courtesy P. Dorlis

Further developments: Use small but ultracold quantum sensors (acoustic resonators @ temperature close to quantum ground state) - sensitive to single graviton flcts? (quantum jumps) → weakness of gravity → advantage - analogies with photoelectric effect

Tobar, Manikandan, Beitel, Pikovski
Nat. Comm. 15, 7229 (2024)

Are these effect distinguishable from classical GW ?

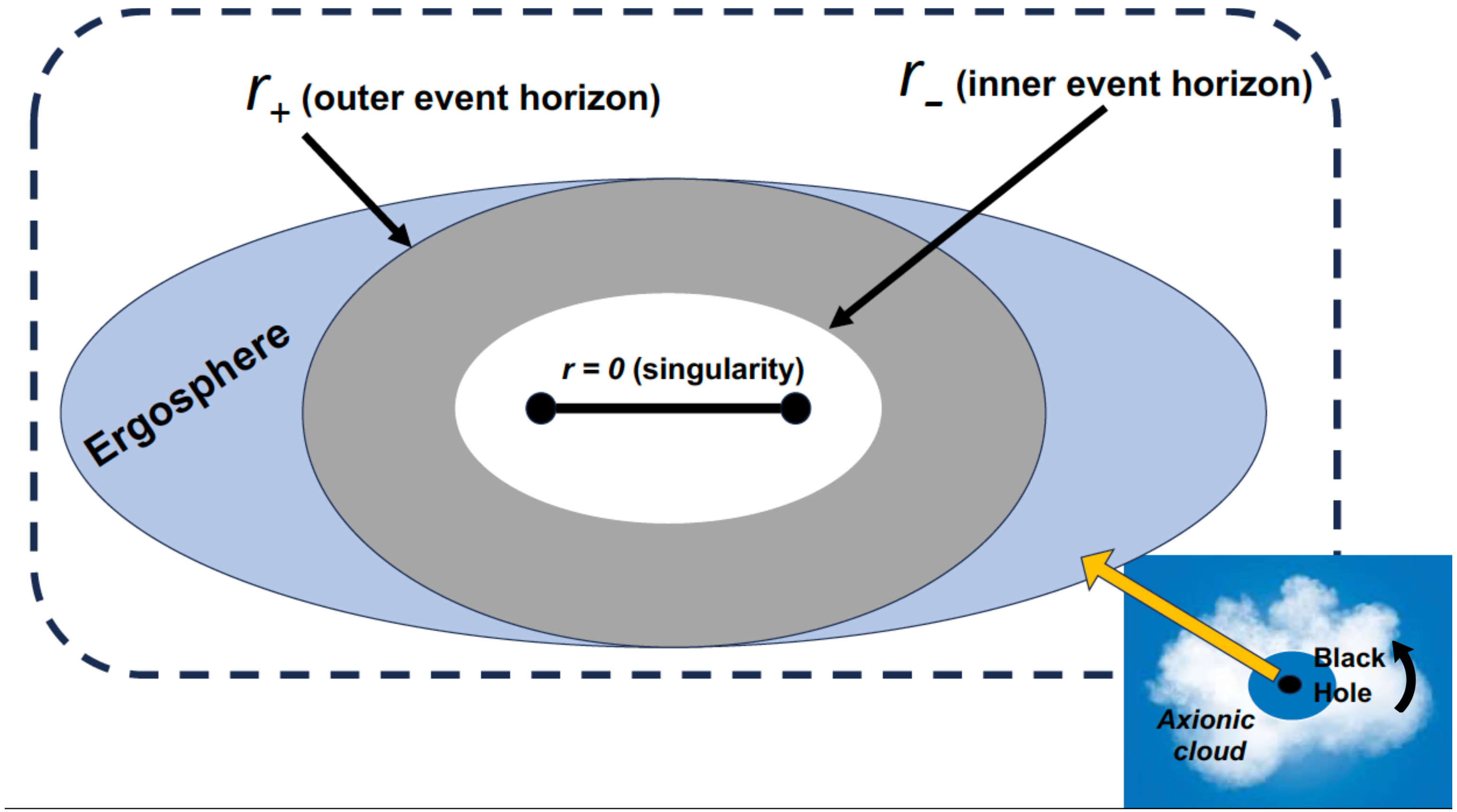
D. Carney, V. Domcke, N. L. Rodd,
PRD 109, 044009 (2024)

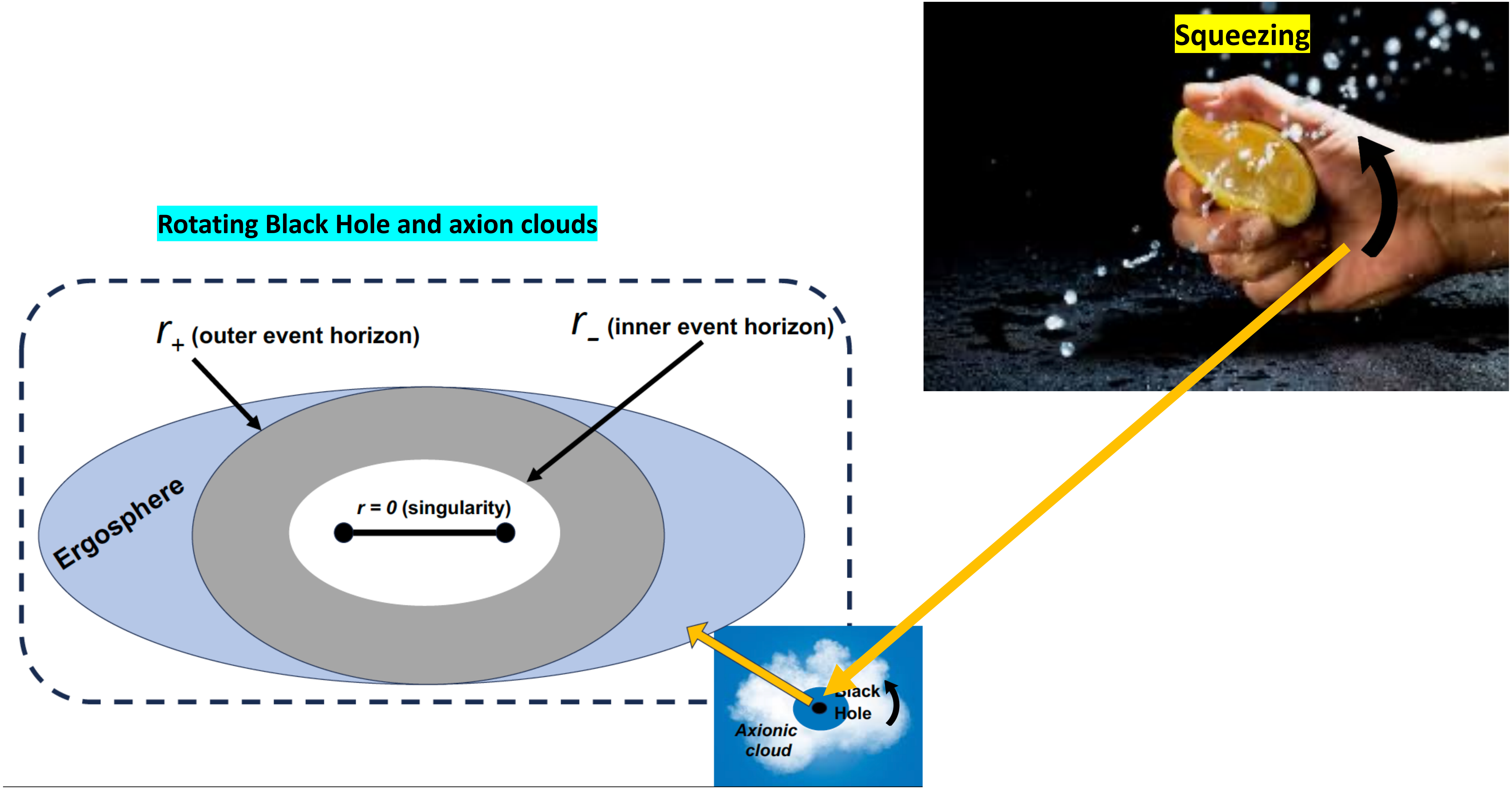
We look for astrophysical situations in which **polarization entangled** states of **quantum gravitons** are produced in **large numbers**, hence leading to **significant enhancement over** the **single-graviton-state** case

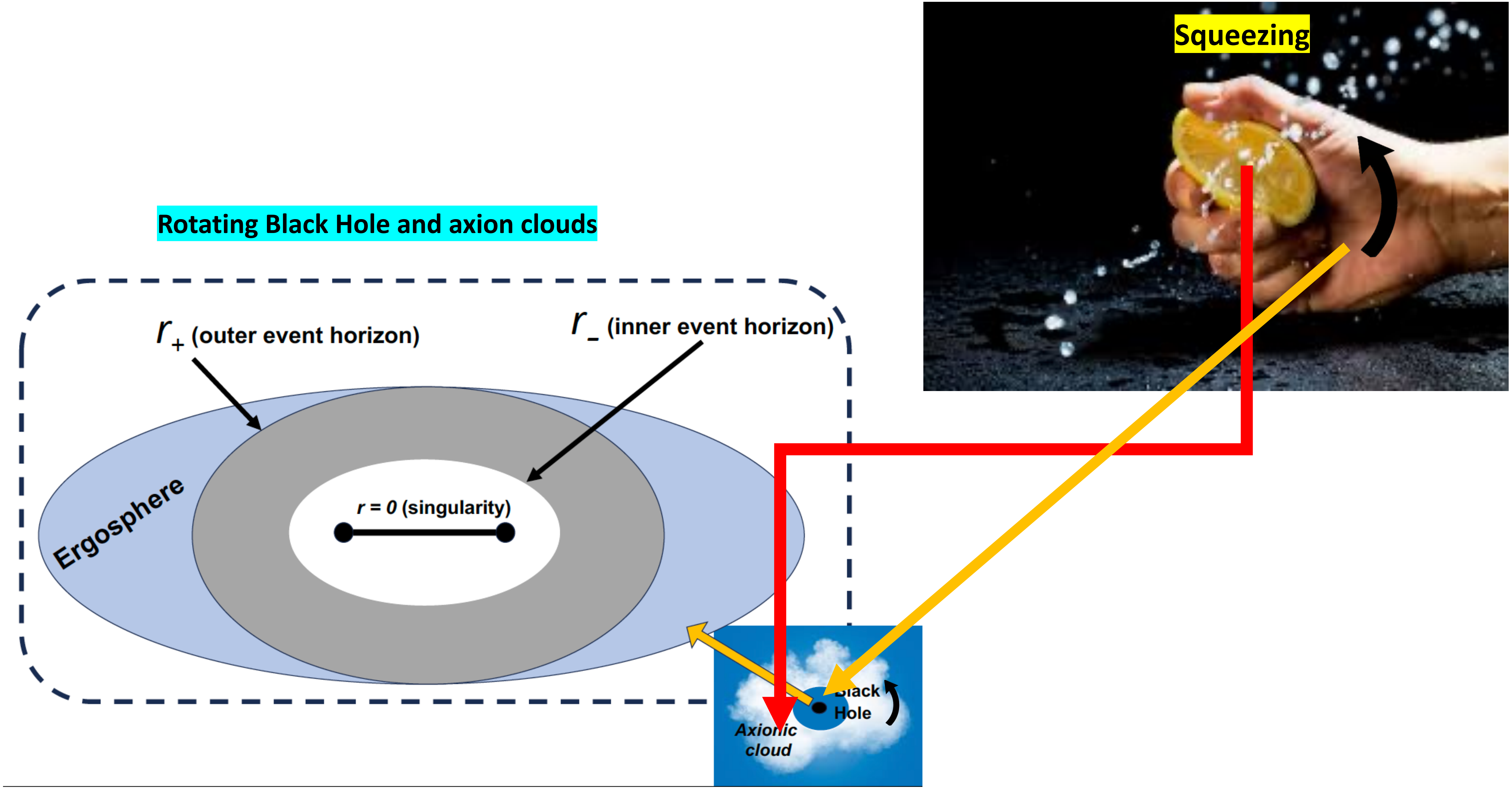
Where are such situations realized?

Dorlis, Mavromatos, Sarkar, and Vlachos,
Phys. Rev. Lett. 135, 151501 (2025);
PRD 113 (2026), 026023
e-Print: 2605.14797 [gr-qc]
(IJMPD, 3rd Award 2026 GRF Essays on Gravitation)

Rotating Black Hole and axion clouds




Squeezing
Rotating Black Hole and axion clouds
r_+ (outer event horizon)
r_- (inner event horizon)
Ergosphere
r = 0 (singularity)
Hole
Axionic cloud



Rotating Black Hole and axion clouds
Squeezing
r_+ (outer event horizon)
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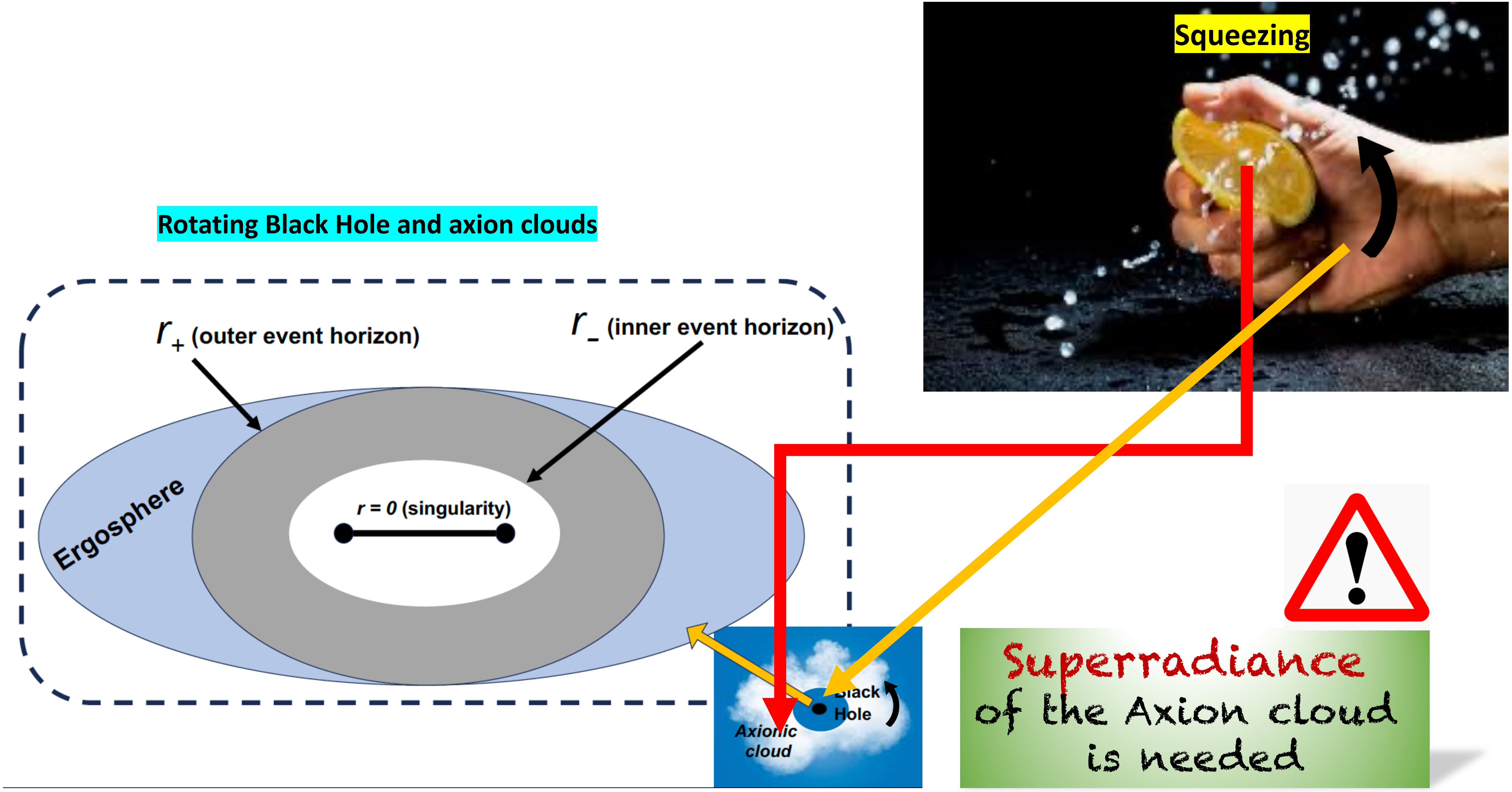
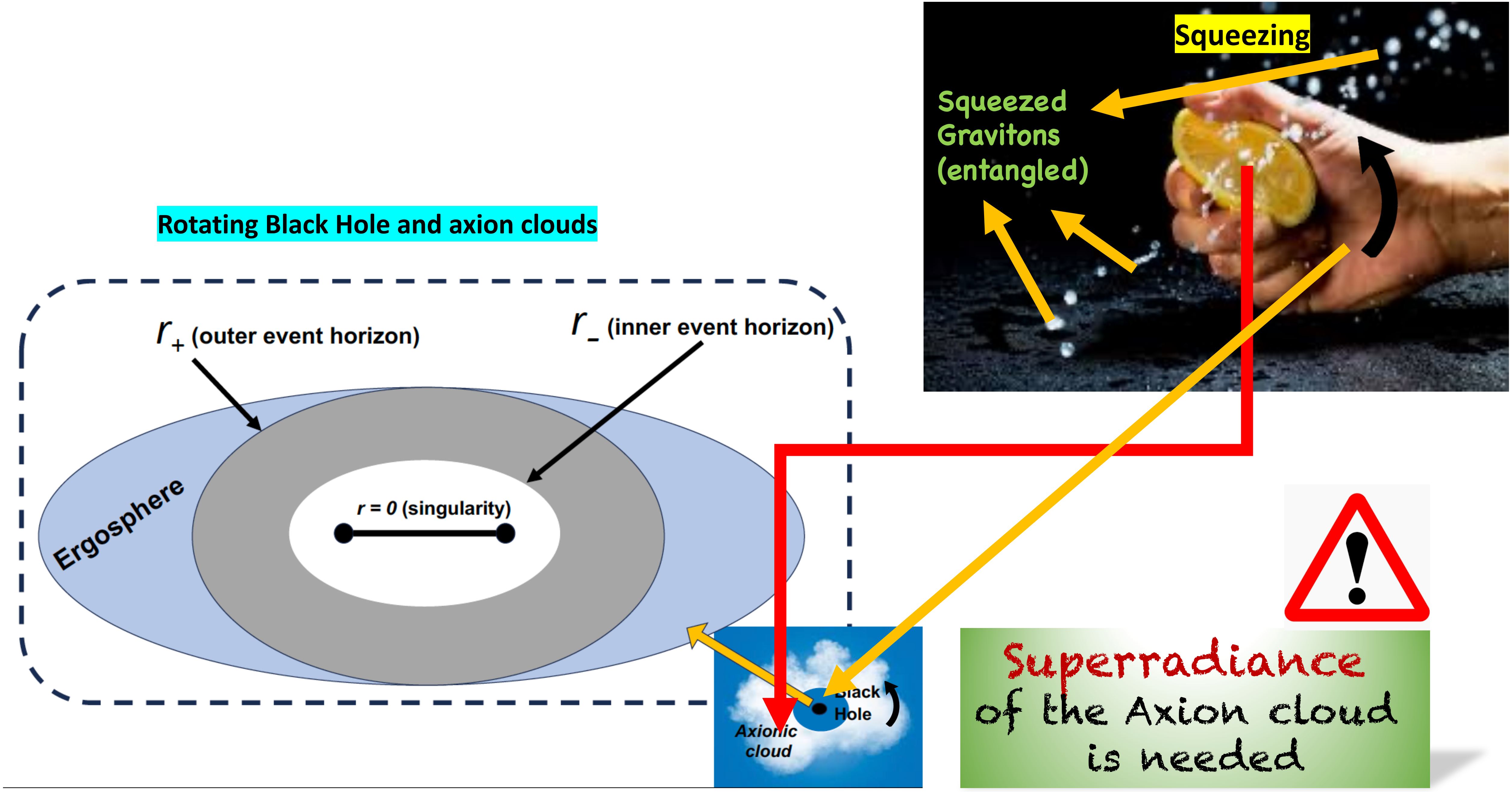


Squeezing
Rotating Black Hole and axion clouds
r_+ (outer event horizon)
r_- (inner event horizon)
Ergosphere
r = 0 (singularity)
Black Hole
Axionic cloud
Superradiance
of the Axion cloud
is needed

Rotating Black Hole and axion clouds

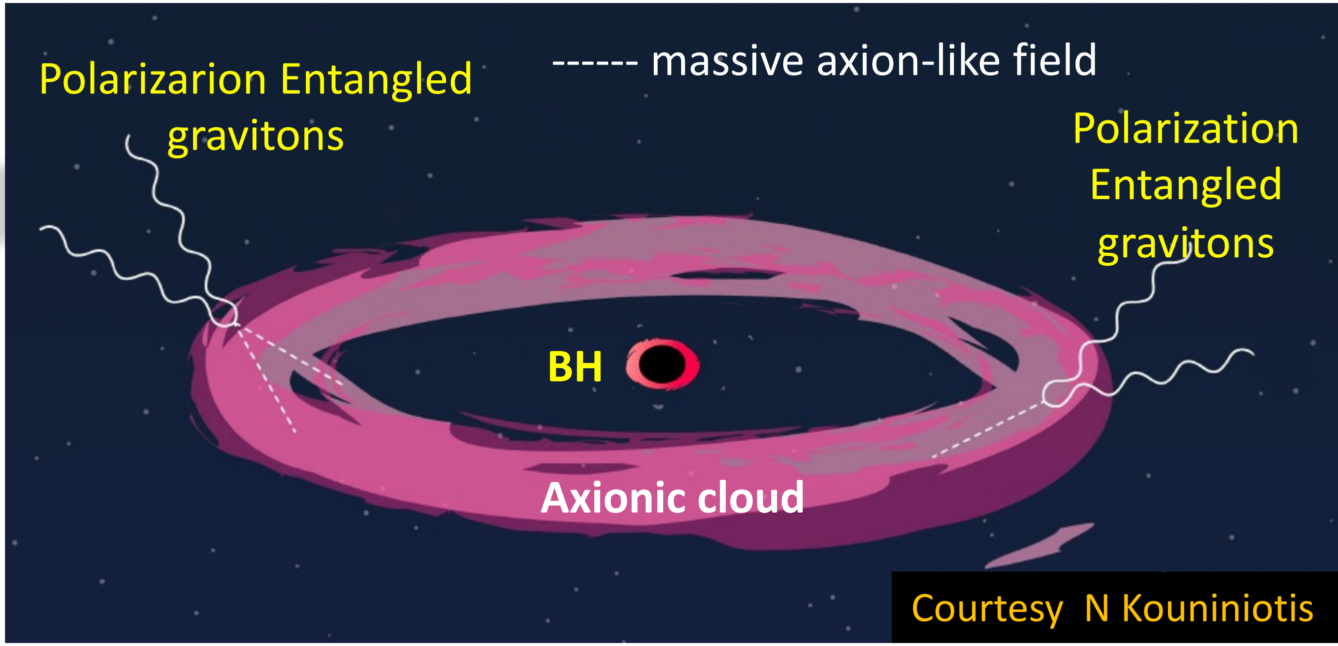


Production of Squeezed Graviton States

Dorlis, NEM, Sarkar, Vlachos,
Phys.Rev.Lett. 135 (2025) 15, 151501
e-Print: 2507.01689 [gr-qc]
& companion long article
Phys. Rev. D 113 (2026), 026023
e-Print: 2507.23475 [gr-qc]

The Ingredients:

- Rotating Black Holes (BH)
- Massive-Axion Superradiant Clouds (kind of condensates in the surrounding area, outside the outer horizons) of finite life time



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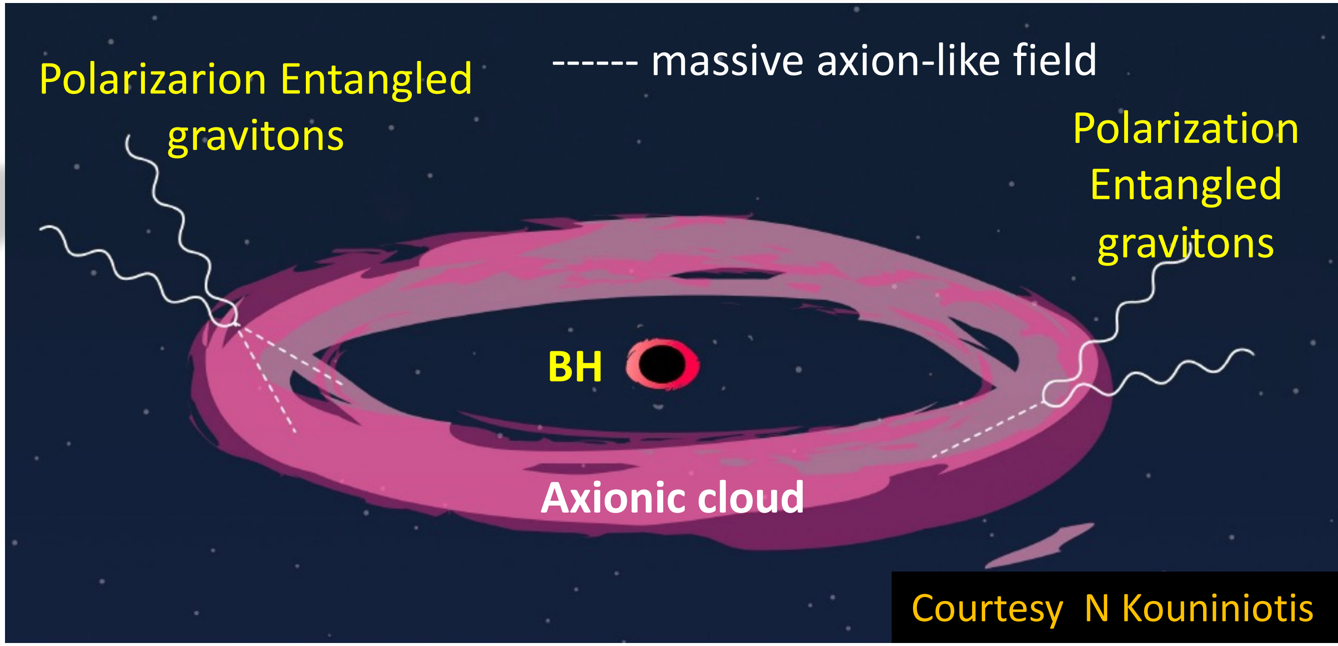
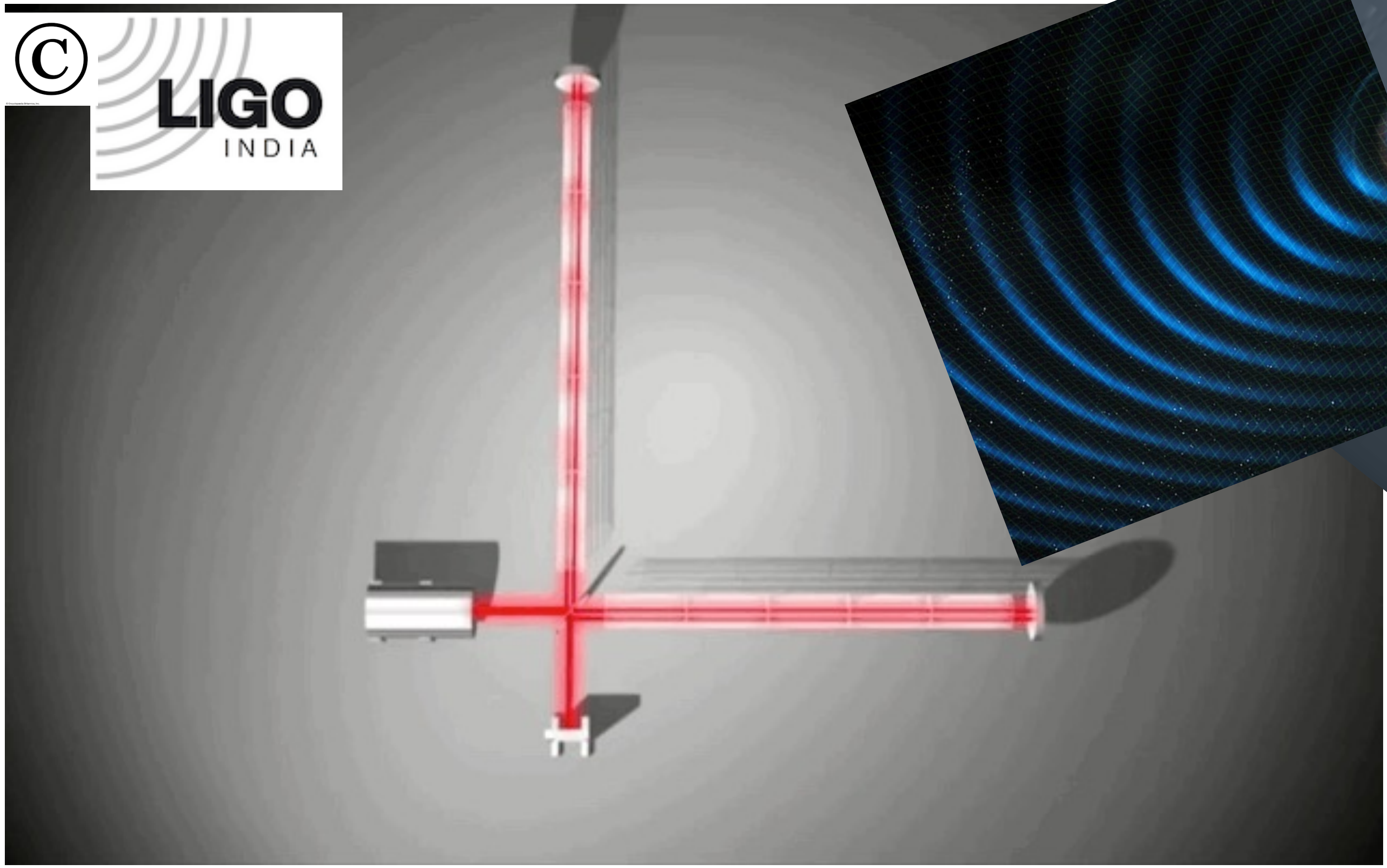
The Ingredients:

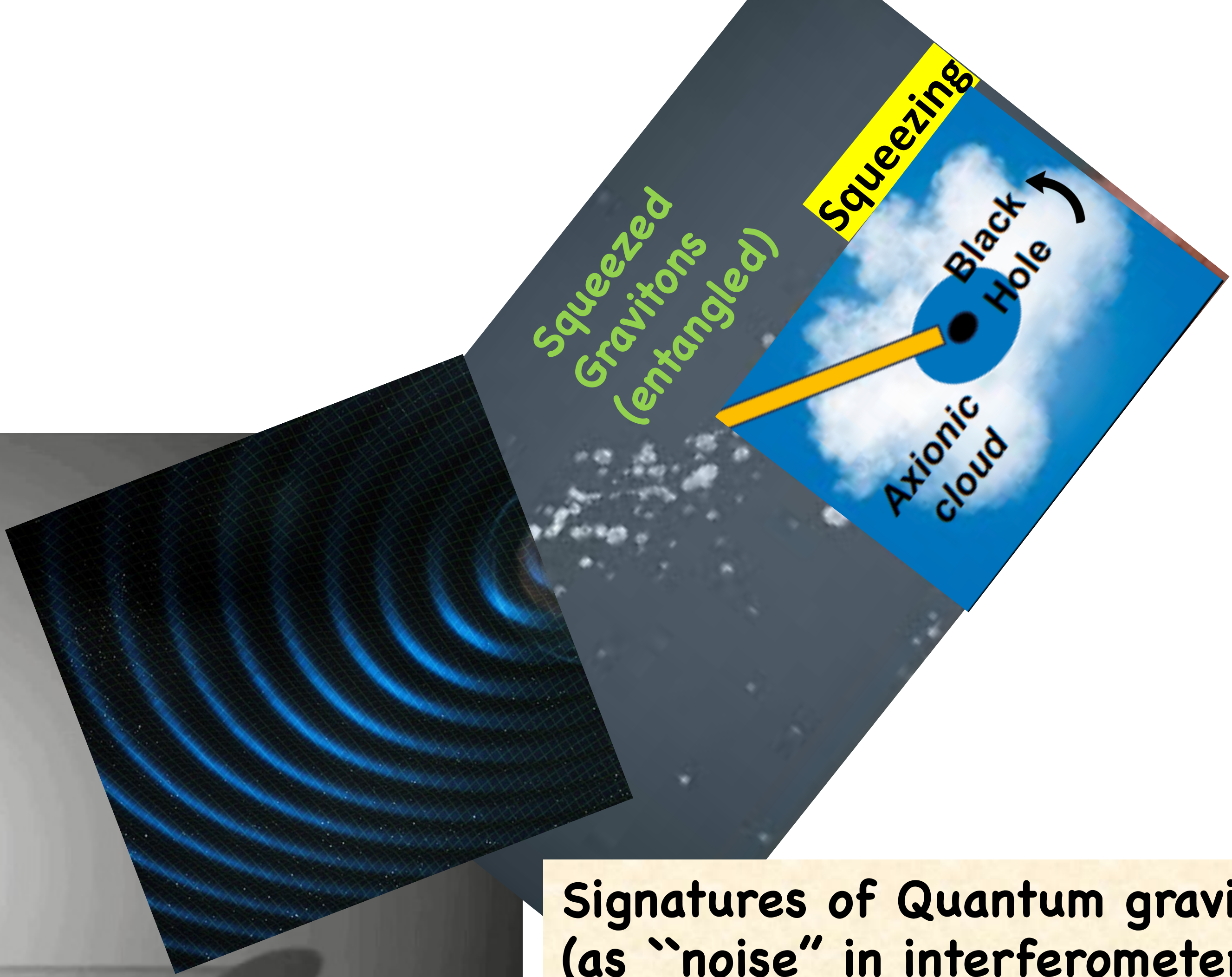
- Rotating Black Holes (BH)
- Massive-Axion Superradiant Clouds (kind of condensates in the surrounding area, outside the outer horizons) of finite life time

Macroscopic number of (massive) Axions
→ Large number of Squeezed Gravitons
→ Enhanced signal of QG compared to single graviton searches



Detection @ Interferometers *etc*

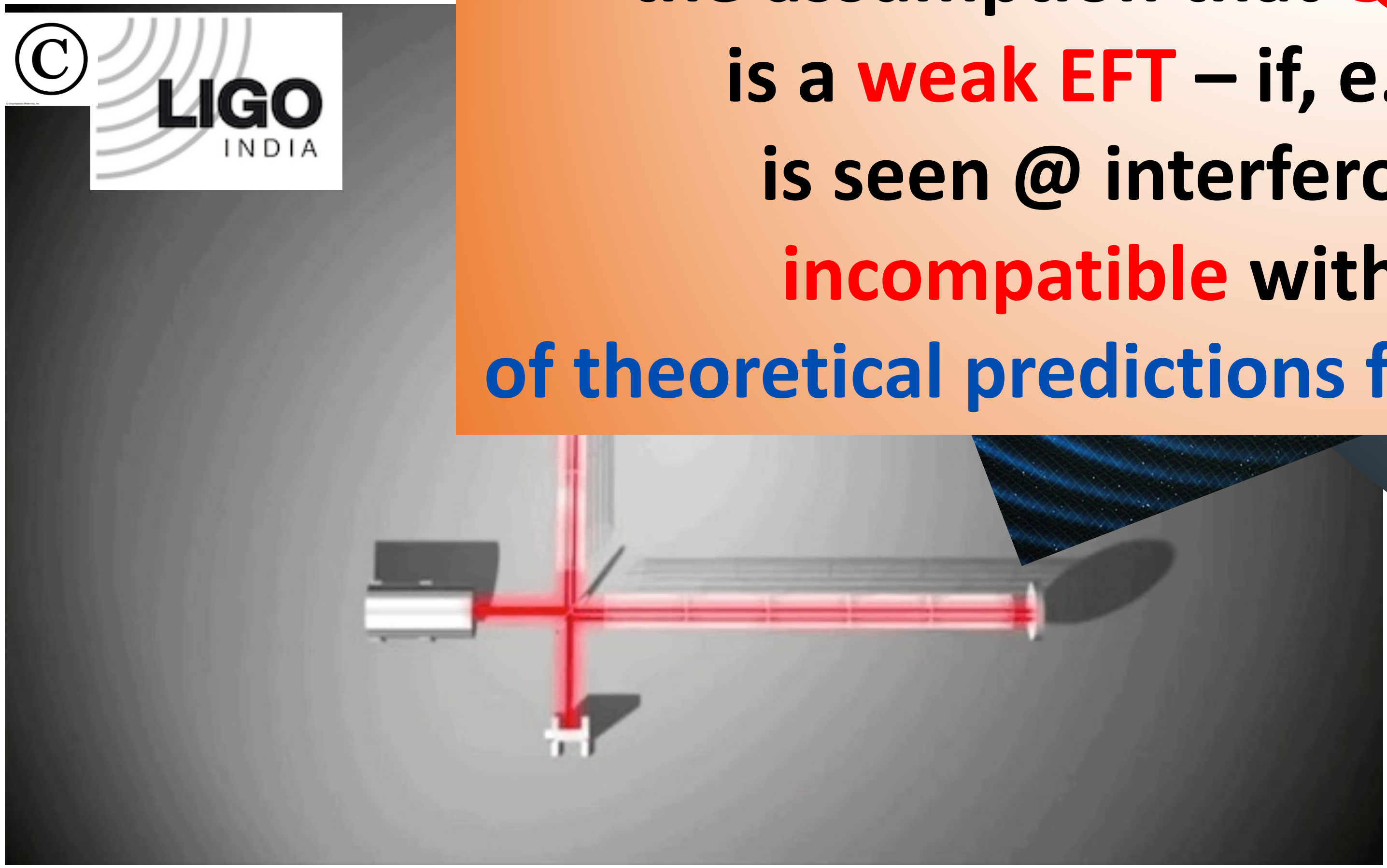


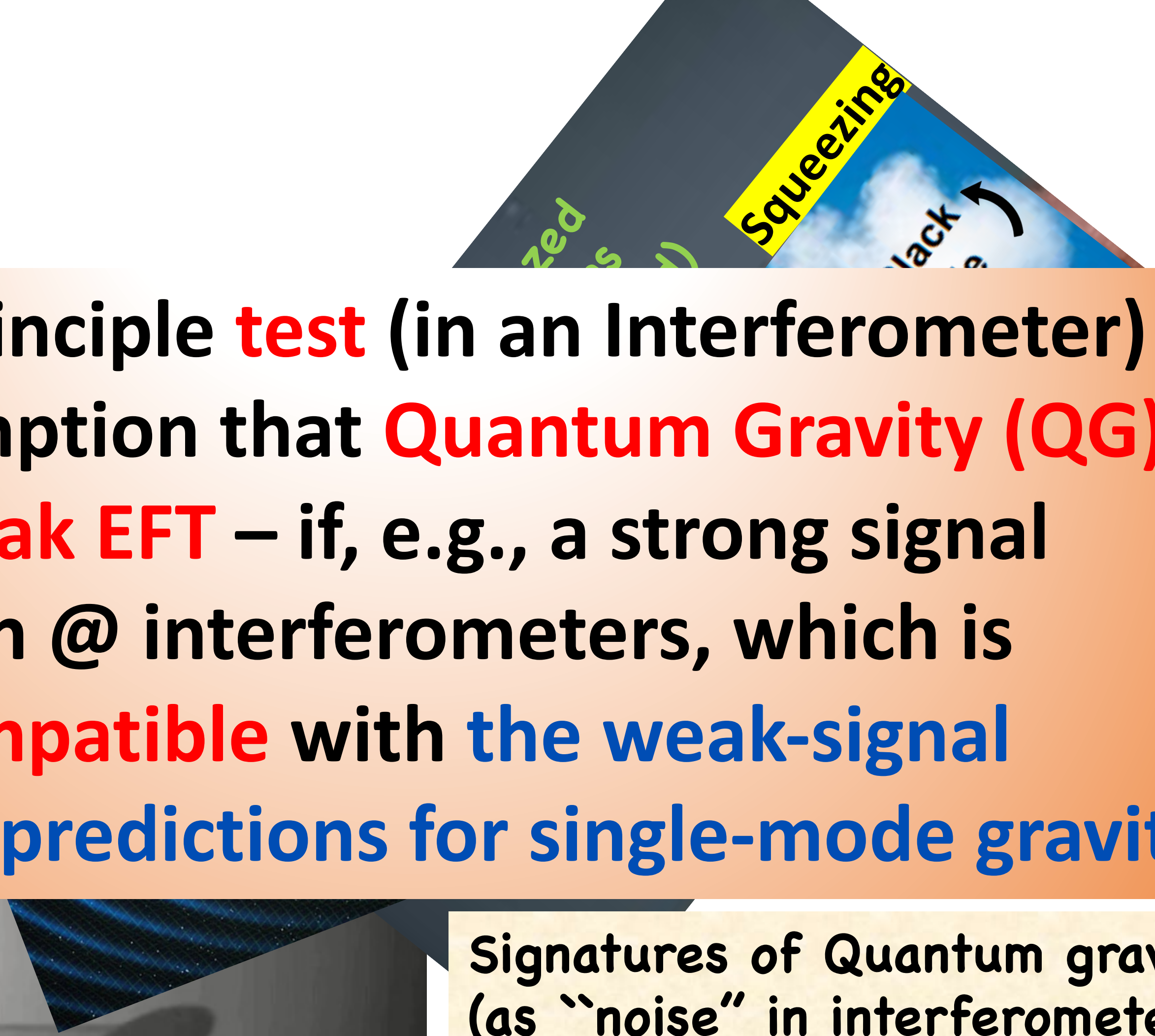


Signatures of Quantum gravity?
(as ``noise" in interferometer)

Testing the effective field theory
Nature of Gravity ?

Detection @





Can in principle test (in an Interferometer) the assumption that Quantum Gravity (QG) is a weak EFT – if, e.g., a strong signal is seen @ interferometers, which is incompatible with the weak-signal of theoretical predictions for single-mode gravitons

Signatures of Quantum gravity?
(as ``noise" in interferometer)

Testing the effective field theory
Nature of Gravity ?

P. Dorlis, N. E. Mavromatos, S. Sarkar, and S.-N. Vlachos,
Phys. Rev. Lett. 135, 151501 (2025]; PRD113 (2026), 026023

Squeezing of Gravitons
in
weak Quantum Gravity EFT:
The formalism

GRAVITATIONAL CHERN-SIMONS (gCS) THEORY

Jackiw, Pi , PRD 68, 104012 (2003)
Alexander, Yunes, Phys. Rept. 480, 1 (2009)

Background of a Rotating (e.g. Kerr) Black Hole (BH):
Non-trivial gCS term

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \frac{1}{2}(\partial_\mu b)(\partial^\mu b) - \frac{1}{2}\mu_b^2\, b^2 - A\, b\, R_{CS} \right]$$

Massive Axion-like Field

$$\kappa = \sqrt{8\pi G} = M_{\rm Pl}^{-1}$$

$$R_{CS} = \frac{1}{2} R^\mu{}_{\nu\rho\sigma} \widetilde{R}^\nu{}_\mu{}^{\rho\sigma} \neq 0$$

(In rotating BH bkgrnd)

$$\widetilde{R}_{\alpha\beta\gamma\delta} = \frac{1}{2} R_{\alpha\beta}{}^{\rho\sigma} \varepsilon_{\rho\sigma\gamma\delta}$$

Dual Riemann in (3+1) dimensions

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Jackiw, Pi , PRD 68, 104012 (2003)
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$$\kappa = \sqrt{8\pi G} = M_{\rm Pl}^{-1}$$

Massive Axion-like Field

NB:

$$G_{\mu\nu} = \kappa^2 T^b_{\mu\nu} + 4\kappa^2 A C_{\mu\nu}$$

$$C_{\mu\nu} = -\frac{1}{2}\nabla^\alpha\left[(\nabla^\beta b)\tilde{R}_{\alpha\mu\beta\nu} + (\nabla^\beta b)\tilde{R}_{\alpha\nu\beta\mu}\right]$$

Cotton-like tensor

$$\nabla_\mu C^{\mu\nu} = -\frac{1}{4}(\nabla^\nu b)\, R_{CS} \Rightarrow \nabla_\mu T_b^{\mu\nu} = A\frac{1}{4}(\nabla^\nu b)\, R_{CS}$$

$$R_{CS} = \frac{1}{2} R^\mu{}_{\nu\rho\sigma}\widetilde{R}^\nu{}_\mu{}^{\rho\sigma} \neq 0$$

(In rotating BH bkgrnd)

No problem with diffeomorphisms → Exchange of energy between b & gravity (CS anomaly term), unitarity OK in EFT framework (with Planck-scale UV cutoff)

GRAVITATIONAL CHERN-SIMONS (gCS) THEORY

Jackiw, Pi , PRD 68, 104012 (2003)
Alexander, Yunes, Phys. Rept. 480, 1 (2009)

Background of a Rotating (e.g. Kerr) Black Hole (BH):
Non-trivial gCS term

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \frac{1}{2}(\partial_\mu b)(\partial^\mu b) - \frac{1}{2}\mu_b^2\, b^2 - A\, b\, R_{CS} \right]$$

Why massive axions?

QCD axions,

String theory axions ...

Massive Axion-like Field

$$\kappa = \sqrt{8\pi G} = M_{\rm Pl}^{-1}$$

$$R_{CS} = \frac{1}{2} R^\mu{}_{\nu\rho\sigma} \widetilde{R}^\nu{}_\mu{}^{\rho\sigma} \neq 0$$

(In rotating BH bkgrnd)

$$\widetilde{R}_{\alpha\beta\gamma\delta} = \frac{1}{2} R_{\alpha\beta}{}^{\rho\sigma} \varepsilon_{\rho\sigma\gamma\delta}$$

Dual Riemann in (3+1) dimensions

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Massive Axion-like Field

$$\kappa = \sqrt{8\pi G} = M_{\rm Pl}^{-1}$$

In string theory Axions could be:
(i) string-model independent axions (acquire masses through instanton effects)

Duncan, Kaloper, Olive (1992)

$$A = \frac{1}{48}\sqrt{\frac{2}{3}}\frac{\alpha'}{\kappa}$$

$\alpha' = M_s^{-2}$

string mass scale

(ii) compactification axions

Svrcek, Witten (2006)

A depends on details of compactification

$$R_{CS} = \frac{1}{2} R^\mu{}_{\nu\rho\sigma} \widetilde{R}^\nu{}_\mu{}^{\rho\sigma} \neq 0$$

(In rotating BH bkgrnd)

$$\widetilde{R}_{\alpha\beta\gamma\delta} = \frac{1}{2} R_{\alpha\beta}{}^{\rho\sigma} \varepsilon_{\rho\sigma\gamma\delta}$$

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$$\kappa = \sqrt{8\pi G} = M_{\rm Pl}^{-1}$$

Why massive axions?

e.o.m. *b(x)-field:*

$$\left(\Box - \mu_b^2\right) b(x) = A R_{CS}$$

Axions in the spectrum of string theories (consistent theories of QG)

Non-trivial gCS sources Axions in exterior neighborhood of a rotating BH

Massive Axions are also DM candidates

$$R_{CS} = \frac{1}{2} R^{\mu}{}_{\nu\rho\sigma} \widetilde{R}^{\nu}{}_{\mu}{}^{\rho\sigma} \neq 0$$

(In rotating BH bkgrnd)

$$\widetilde{R}_{\alpha\beta\gamma\delta} = \frac{1}{2} R_{\alpha\beta}{}^{\rho\sigma} \varepsilon_{\rho\sigma\gamma\delta}$$

: Dual Riemann in (3+1) dimensions

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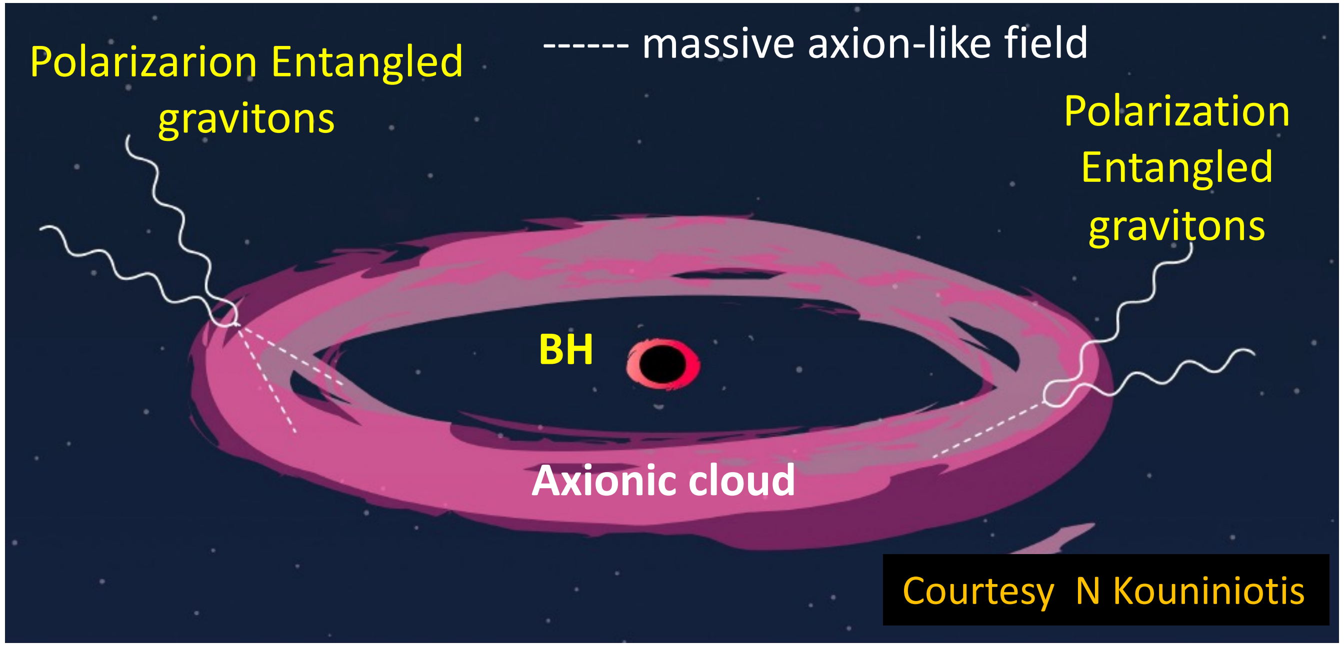
$$\kappa = \sqrt{8\pi G} = M_{\rm Pl}^{-1}$$

Why massive axions?

Moreover

$$\left(\Box - \mu_b^2\right) b(x) = A R_{CS}$$

Axions in the BH background can form clouds (\`\`condensate-like'' structures) & exhibit SUPERRADIANCE



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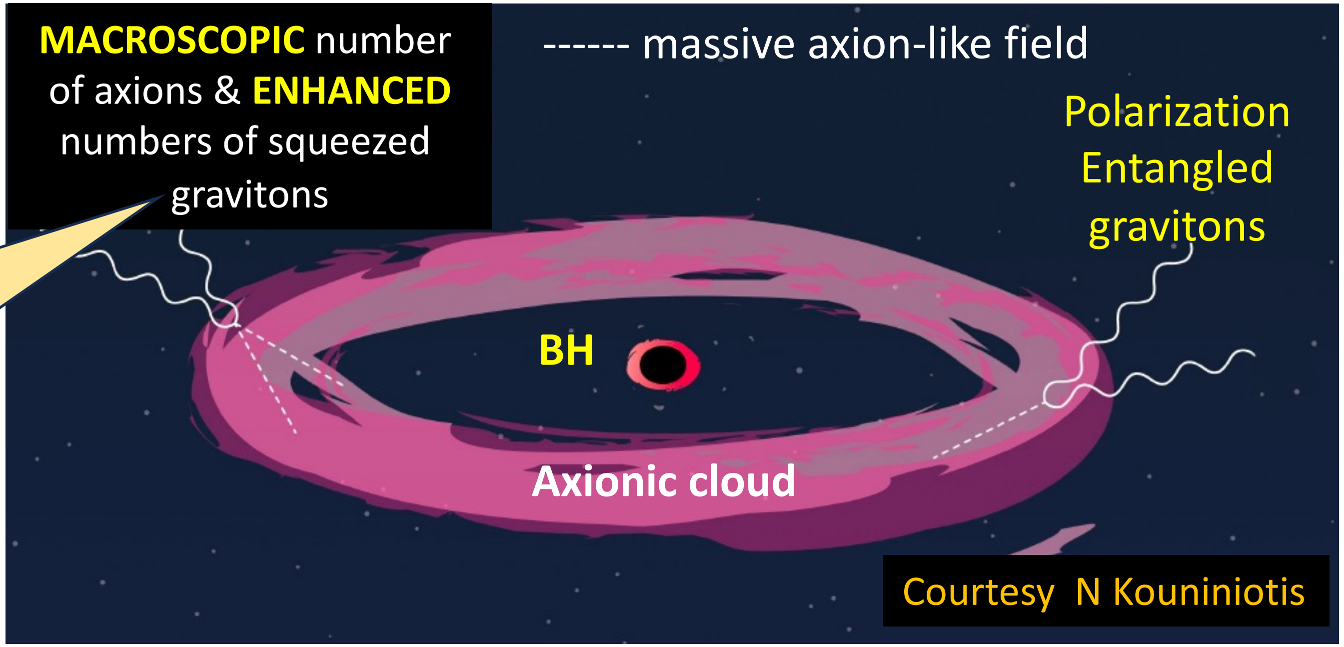

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Massive Axion-like Field

Why massive axions?

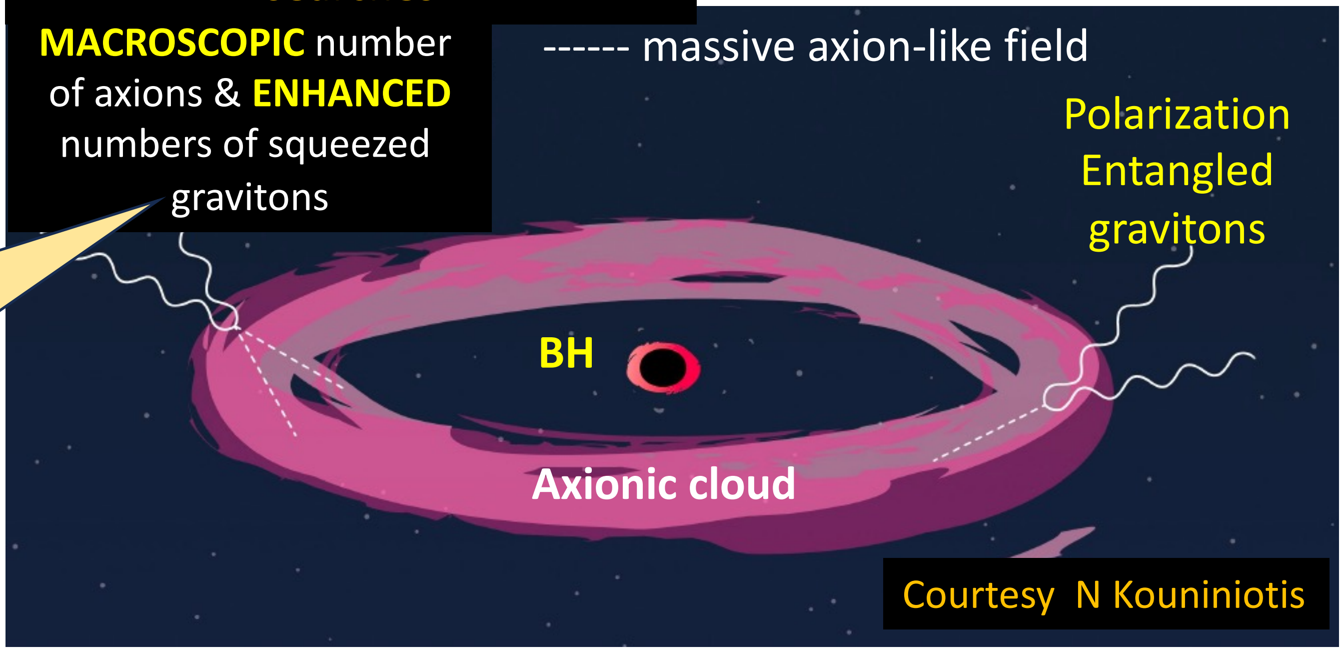
Much better phenomenological prospects than in single-graviton searches

Moreover

$$\left(\Box - \mu_b^2\right) b(x) = A R_{CS}$$

Axions in the BH background can form clouds (\`\`condensate-like'' structures) & exhibit SUPERRADIANCE

MACROSCOPIC number of axions & ENHANCED numbers of squeezed gravitons



Courtesy N Kouniniotis

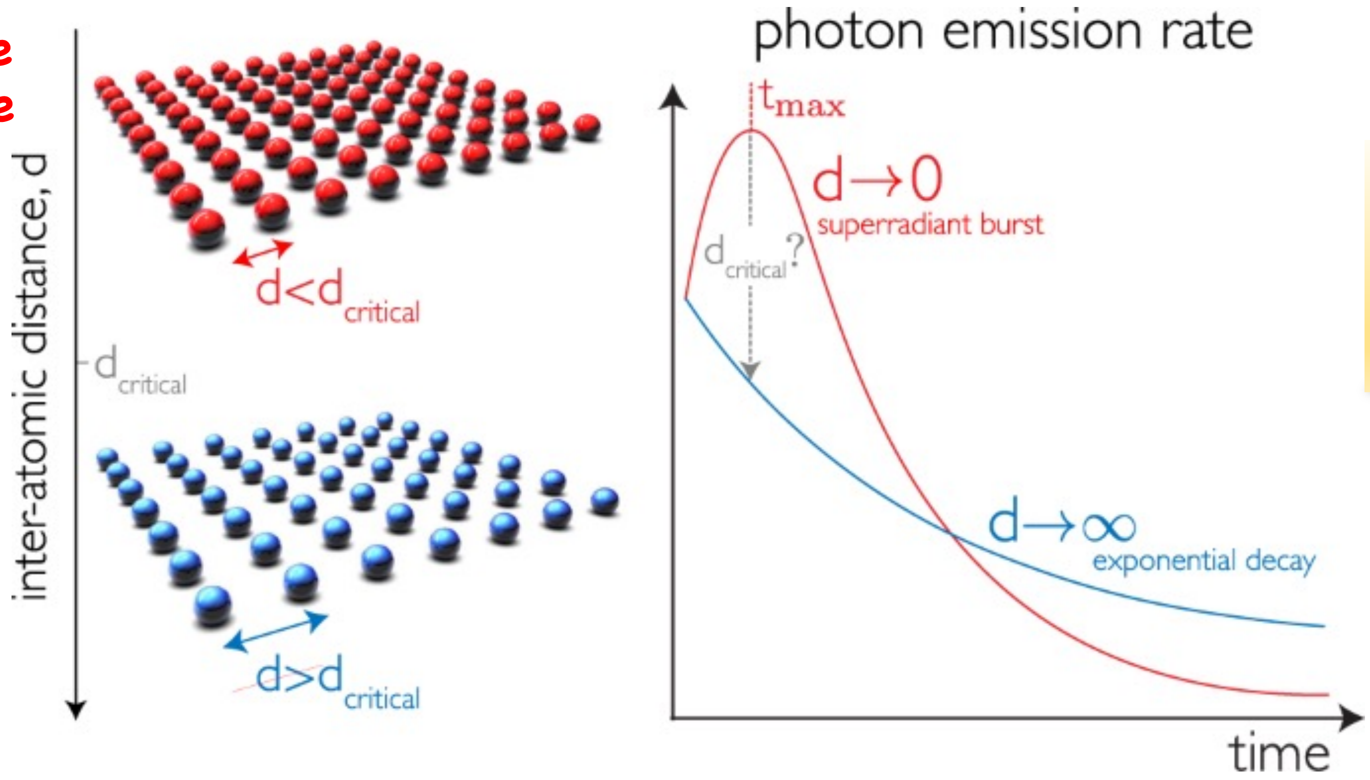
Black-Hole-Axionic-Cloud Superradiance

NB:

Superradiance in Optics

In Optics: **Superradiance** is a phenomenon where **many excited atomic emitters,** interacting with a **common light field**, **release** their **energy collectively**, which results in the emission of a synchronized, intense burst of light, leading to an **amplified emission rate,** provided the wavelength of the light field is **much larger** than the separation of the emitters

emitters are close enough to act like a giant antenna



NB: not necessarily associated with instabilities

NB:

Rotating-Bodies Electromagnetic Superradiance

Y.B. Zel'dovich, ZhETF Pis ma Redaktsiiu 14, 270 (1971)

Superradiance also occurs in situations in which the amplitude of the transmitted electromagnetic wave in interaction with a rotating object is larger than that of the incident wave, thus leading to amplification, provided the radiation is prepared in an appropriate angular momentum state;
e.g. the region around the equator of a spinning metal sphere is expected to throw off electromagnetic radiation along the tangential direction.

Condition

$$\boxed{\omega_r < m\ \Omega}$$

frequency of rotating body

radiation frequency

component of angular momentum along axis of rotation

The rotating-body superradiance is not necessarily associated with an instability.

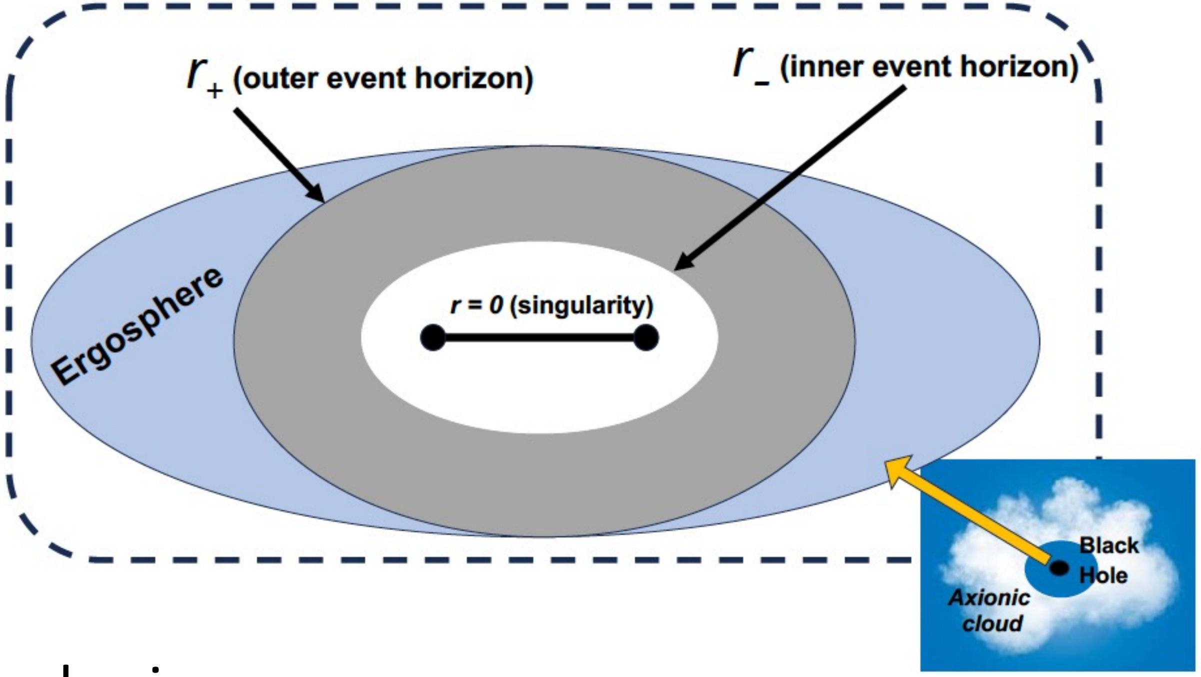
Zel'dovich suggestion: the case of a **spinning gravitational mass**, such as a **Kerr black hole**, ought to produce similar coupling effects, and **radiate in an analogous way**.

BH Axionic-Cloud Superradiance

Detweiler, PRD 22, 2323 (1980)
Britto, Cardoso, Pani, Lect. Notes Phys. 906, 1 (2015)

In our **gravitational context, there is a ``runaway superradiance'' :**
Particles and electromagnetic radiation scattering from a **spinning black hole**, can gain energy and angular momentum in the process.

If this radiation is reflected back at the black hole, a runaway explosive process (sometimes called a "black-hole bomb") could develop. This requires the presence of a light (pseudo) scalar field, in our case the massive axion b(x).
In a rotating BH background, this field exhibits an **exponentially growing mode instability.**
Upon extracting significant amounts of energy from the spinning BH, the axions form **dense clouds** of finite life time T, a kind of ``**condensate**'' in the exterior region of the outer horizon, extending to **macroscopically large** distances **from the horizon.**



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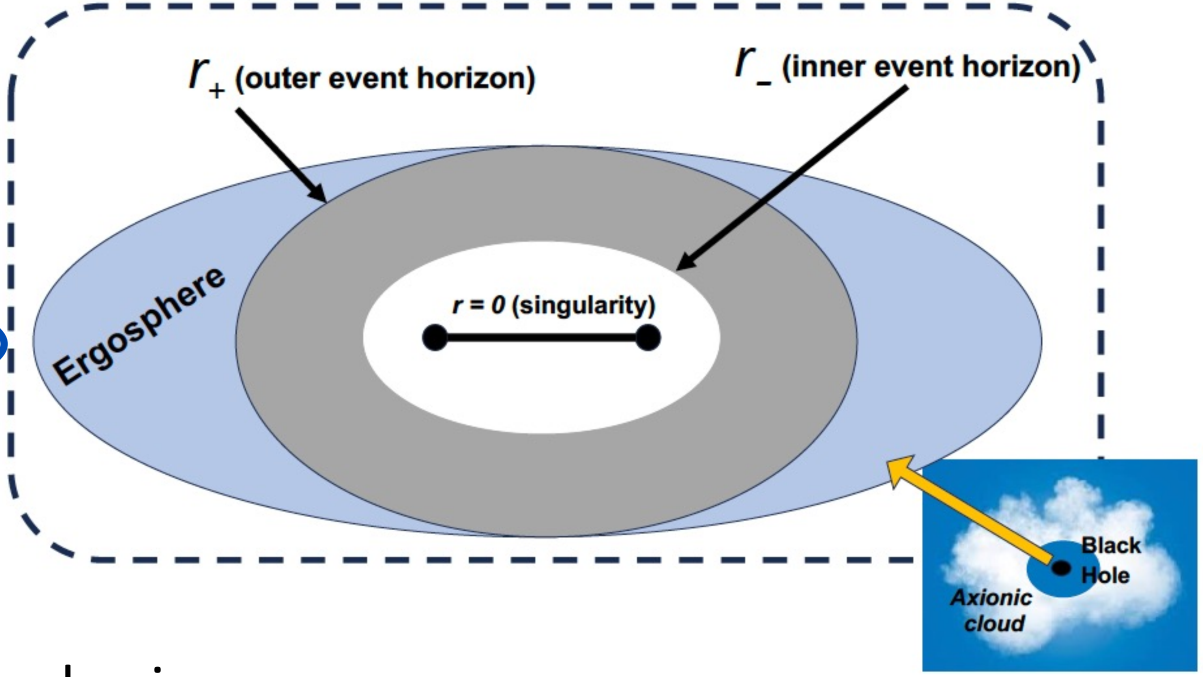


P. Dorlis, N. E. Mavromatos, S. Sarkar, and S.-N. Vlachos,
Phys. Rev. Lett. 135, 151501 (2025]; PRD113 (2026), 026023

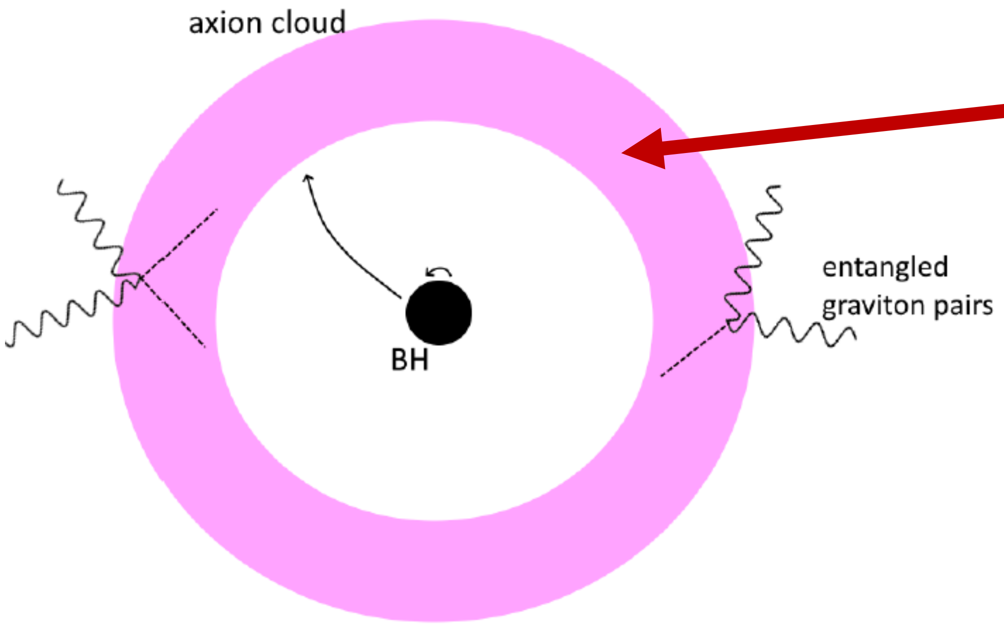
Production of Multi-mode Squeezed Graviton states and analogies with Quantum Optics

View weak Gravity as an EFT (non renormalizable, but for a low energy theory, compared to Planck scales, this is OK)

$$g_{\mu\nu} \to g_{\mu\nu} + \kappa h_{\mu\nu} \,,$$
$$g^{\mu\nu} \to g^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2 h^{\mu\alpha} h_\alpha^\nu$$

background metric

Expand up to & including 2nd order weak tensor perturbations $h_{\mu\nu}$



Regions of interest for the production of Squeezed Gravitons → deep in the cloud far away from the horizon

In those regions, Background can be approximated by Minkowski

$$g_{\mu\nu} \simeq \eta_{\mu\nu}$$

Canonical Quantization of Gravitational-Wave (GW) tensor perturbations in Chern-Simons Gravity

The GW action:

$$S_{GW} = \frac{1}{4} \sum_{\lambda=L,R} \int dt \int d^3\vec{k} \left(|\dot{h}_{\lambda,\vec{k}}|^2 - k^2 |h_{\lambda,\vec{k}}|^2 \,.\right)$$

Graviton quantum operators

$$\hat{h}_{ij}(t,\vec{x}) = \int \frac{d^3\vec{k}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\Omega_k}} \sum_{\lambda=L,R} \left[e^{(\lambda)}_{ij,\vec{k}} \hat{\alpha}^\dagger_{\lambda,\vec{k}} \; e^{-ik\cdot x} + \text{h.c.} \right] \qquad \Omega_k \equiv k$$

$$\left[\hat{\alpha}_{\lambda,\vec{k}} \,,\; \hat{\alpha}^\dagger_{\lambda',\vec{k}'} \right] = \delta_{\lambda,\lambda'} \; \delta^{(3)}\left(\vec{k} - \vec{k}'\right)$$

$$e^{(L,R)}_{ij}(\vec{k}) = e^{(L,R)}_{ij}(-\vec{k}) \;,$$

$$e^{(L)}_{ij}(\vec{k}) \; e^{(L)\,ij}(\vec{k}) = e^{(R)}_{ij}(\vec{k}) \; e^{(R)\,ij}(\vec{k}) = 0 \;,$$

$$e^{(L)}_{ij}(\vec{k}) e^{(R)\,ij}(\vec{k}) = 2 \;, \qquad \text{with } e_3(\vec{k}) = \vec{k}/|\vec{k}|$$

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$$\left[\hat{\alpha}_{\lambda,\vec{k}}\ ,\ \hat{\alpha}^\dagger_{\lambda',\vec{k}'}\right] = \delta_{\lambda,\lambda'}\ \delta^{(3)}\left(\vec{k}-\vec{k}'\right)$$

$$\int \frac{d^3\vec{k}}{(2\pi)^{3/2}} \to \frac{1}{V}\sum_{\vec{k}}\ ,\quad \hat{\alpha}_{\lambda,\vec{k}} \to \sqrt{V}\ \hat{\alpha}_{\lambda,\vec{k}} \quad \Rightarrow \quad \boxed{\hat{h}_{ij}(t,\vec{x}) = M_{\rm Pl}\sum_{\vec{k},\lambda} f_k\left[e^{(\lambda)}_{ij}(\vec{k})\ \hat{\alpha}^\dagger_{\lambda,\vec{k}}\ e^{-ik\cdot x} + \text{h.c.}\right]}$$

$$f_k \equiv \frac{\kappa}{\sqrt{2V\Omega_k}}$$

GW Hamiltonian operator

$$\boxed{\hat{\mathcal{H}}^{(0)}_{GW} = \int d^3\vec{k}\ \Omega_k\sum_{\lambda=L,R}\hat{\alpha}^\dagger_{\lambda,\vec{k}}\hat{\alpha}_{\lambda,\vec{k}}}$$

NB: like collection of harmonic oscillators

Chern-Simons Gravity EFT up to second order in tensor perturbations

$$S=\int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2}-\frac{1}{2}(\partial_\mu b)(\partial^\mu b)-\frac{1}{2}\mu_b^2 b^2-AbR_{\text{CS}}\right]$$

$$g_{\mu\nu}\to g_{\mu\nu}+\kappa h_{\mu\nu},$$

$$g^{\mu\nu}\to g^{\mu\nu}-\kappa h^{\mu\nu}+\kappa^2 h^{\mu\alpha}h^\nu_\alpha,$$

$$\delta\mathcal{L}^{(2)}_{\text{EH}}=\frac{1}{4}\sqrt{-g}G_{\mu\nu}h^\mu_\alpha h^{\alpha\nu}-\frac{1}{4}\sqrt{-g}hh^{\alpha\beta}G_{\alpha\beta}+\frac{1}{2}\sqrt{-g}\left[-\frac{1}{8}h^2R-\frac{1}{2}R^\alpha{}_{\lambda\beta\gamma}h^\gamma_\alpha h^{\lambda\beta}+\frac{1}{2}\nabla^\gamma h_{\alpha\gamma}\nabla_\beta h^{\alpha\beta}+\frac{1}{4}\nabla_\beta h\nabla^\beta h\right.$$
$$\left.-\frac{1}{2}\nabla_\beta h\nabla_\alpha h^{\alpha\beta}-\frac{1}{4}\nabla_\gamma h_{\alpha\beta}\nabla^\gamma h^{\alpha\beta}\right],$$

$$\delta\mathcal{L}^{(2)}_{\text{matter}}=\sqrt{-g}\left[\frac{\kappa}{2}T_{\mu\nu}h^{\mu\nu}+\frac{\kappa^2}{4}hh_{\alpha\beta}\nabla^\alpha b\nabla^\beta b+\frac{\kappa^2}{4}\left(h^\mu_\alpha h^{\alpha\nu}-\frac{1}{2}hh^{\mu\nu}\right)g_{\mu\nu}\mathcal{L}_{\text{matter}}-\frac{\kappa^2}{2}h_{\alpha\beta}h^\beta_\mu\nabla^\alpha b\nabla^\mu b\right]$$

$$T_{\mu\nu}=\partial_\mu b\partial_\nu b-\frac{1}{2}g_{\mu\nu}(\eta^{\rho\sigma}\partial_\rho b\partial_\sigma b-\mu^2b^2).$$

Gauge (general coordinate) invariance of the 2nd order Lagrangian

$$b \to b + \xi_\alpha \partial^\alpha b$$

$$\kappa h_{\mu\nu} \to \kappa h_{\mu\nu} + (g_{\alpha\nu} + \kappa h_{\alpha\nu})\nabla_\mu \xi^\alpha + (g_{\mu\alpha} + \kappa h_{\mu\alpha})\nabla_\nu \xi^\alpha + \kappa \xi^\alpha \nabla_\alpha h_{\mu\nu},$$

Gauge fixing: Transverse – Traceless gauge $\quad h^{0\mu} = 0,\ h = 0, \text{ and } \nabla_\mu h^{\mu\nu} = 0,$

Simplificatio., upon using also background grvitational eqs of motion $\quad G_{\mu\nu} = 0 \quad$:

$$\delta \mathcal{L}^{(2)}_{\rm EH} = -\frac{1}{4\kappa^2}\sqrt{-g}\left[\frac{1}{2}\nabla_\gamma h_{\alpha\beta}\nabla^\gamma h^{\alpha\beta} + R^\alpha{}_{\lambda\beta\gamma} h^\gamma_\alpha h^{\lambda\beta}\right].$$

$$\delta \mathcal{L}^{(2)}_{\rm matter} = \sqrt{-g}\left[\frac{\kappa}{2} T_{\mu\nu} h^{\mu\nu} - \frac{\kappa^2}{2} h_{\alpha\beta} h^\beta_\mu \nabla^\alpha b \nabla^\mu b\right]$$

$$R_{\rm CS} = \frac{\kappa^2}{2}\epsilon_{\beta\mu\rho\sigma}(\partial_\alpha \partial^\sigma h^\rho_\nu - \partial_\nu \partial^\sigma h^\rho_\alpha)\partial^\mu \partial^\nu h^{\alpha\beta} + \text{higher orders},$$

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$$\eta_{\mu\nu} \to \eta_{\mu\nu} + \kappa h_{\mu\nu}, \quad \kappa h_{\mu\nu} \ll 1$$

$$\mu, \nu = 0, \ldots 3$$

T_{ij} axion matter stress energy tensor

I,j =1,2,3

$$S^{(1)}_{\rm GR} = \frac{\kappa}{2}\int d^4x h_{ij} T^{ij},$$

$$S^{(2)}_{\rm GR} = -\frac{\kappa^2}{2}\int d^4x h_{im} h^m_j \partial^i b \partial^j b,$$

$$S^{(2)}_{CS} = -A\kappa^2 \int d^4x b(x) \epsilon_{ijk}\left(\partial_l \partial^k h^j_m \partial^m \dot{h}^{li} + \ddot{h}^{li} \partial^k \dot{h}^j_l - \partial_m \partial^k h^j_l \partial^m \dot{h}^{li}\right)$$

2nd - order gauge invariant EFT - gauge fix : TT-gauge (TT = Transverse-Traceless)

$$\nabla_\mu h^{\mu\nu}_{\rm TT} = 0,$$

$$h^{0\mu}_{\rm TT} = h_{{\rm TT}\,\mu}{}^{\mu} = 0$$

G. 't Hooft , M. J. G. Veltman,
Ann. Inst. H. Poincare A Phys. Theor. 20, 69 (1974)

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Pertain to contributions as in conventional GR

$$S^{(2)}_{CS} = -A\kappa^2 \int d^4x b(x) \epsilon_{ijk}\left(\partial_l \partial^k h^j_m \partial^m \dot{h}^{li} + \ddot{h}^{li}\partial^k \dot{h}^j_l - \partial_m \partial^k h^j_l \partial^m \dot{h}^{li}\right)$$

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Pertains to CS anomalous terms contributions

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produces coherent gravitons

$$S^{(2)}_{\mathrm{GR}} = -\frac{\kappa^2}{2}\int d^4x h_{im} h^m_j \partial^i b \partial^j b,$$

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$$\mu, \nu = 0, \dots 3$$

$$S^{(1)}_{\mathrm{GR}} = \frac{\kappa}{2} \int d^4x h_{ij} T^{ij},$$

produces coherent gravitons, not of interest here

coherent states have minimal fluctuations,
almost classical states → very weak signal

$$S^{(2)}_{\mathrm{GR}} = -\frac{\kappa^2}{2} \int d^4x h_{im} h^m_j \partial^i b \partial^j b,$$

$$S^{(2)}_{CS} = -A\kappa^2 \int d^4x b(x) \epsilon_{ijk} \left(\partial_l \partial^k h^j_m \partial^m \dot{h}^{li} + \ddot{h}^{li} \partial^k \dot{h}^j_l - \partial_m \partial^k h^j_l \partial^m \dot{h}^{li} \right)$$

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produces coherent gravitons, not of interest here

$$S^{(2)}_{\text{GR}} = -\frac{\kappa^2}{2}\int d^4x h_{im} h^m_j \partial^i b \partial^j b,$$

produce squeezed entangled gravitons, of interest here

$$S^{(2)}_{CS} = -A\kappa^2 \int d^4x b(x) \epsilon_{ijk}\left(\partial_l \partial^k h^j_m \partial^m \dot{h}^{li} + \ddot{h}^{li}\partial^k \dot{h}^j_l - \partial_m \partial^k h^j_l \partial^m \dot{h}^{li}\right)$$

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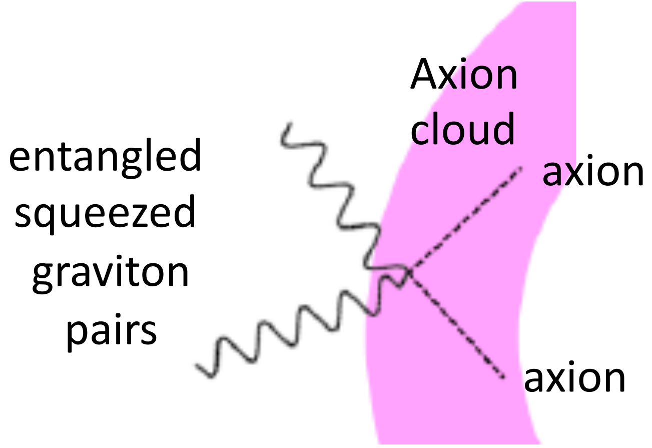
$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \frac{1}{2}(\partial_\mu b)(\partial^\mu b) - \frac{1}{2}\mu_b^2\, b^2 - A\, b\, R_{CS} \right]$$

Quantum Optics Analogue: Spontaneous Four-wave Mixing (SFWM)



$$S^{(1)}_{\mathrm{GR}} = \frac{\kappa}{2} \int d^4x h_{ij} T^{ij},$$

$$S^{(2)}_{\mathrm{GR}} = -\frac{\kappa^2}{2} \int d^4x h_{im} h^m_j \partial^i b \partial^j b,$$

produce squeezed entangled gravitons, of interest here

$$S^{(2)}_{CS} = -A\kappa^2 \int d^4x b(x) \epsilon_{ijk} \left(\partial_l \partial^k h^j_m \partial^m \dot{h}^{li} + \ddot{h}^{li} \partial^k h^j_l - \partial_m \partial^k h^j_l \partial^m \dot{h}^{li} \right)$$

Chern-Simons Gravity EFT up to second order in tensor perturbations

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - \frac{1}{2}(\partial_\mu b)(\partial^\mu b) - \frac{1}{2}\mu_b^2\, b^2 - A\, b\, R_{CS} \right]$$

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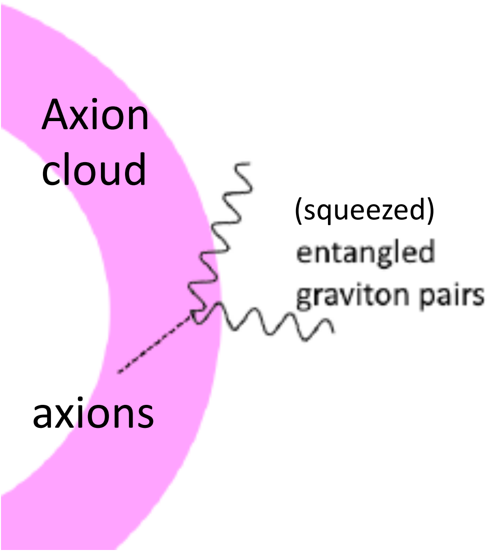
$$S^{(2)}_{CS} = -A\kappa^2 \int d^4x b(x) \epsilon_{ijk} \Big(\partial_l \partial^k h^j_m \partial^m \dot{h}^{li} + \ddot{h}^{li} \partial^k \dot{h}^j_l - \partial_m \partial^k h^j_l \partial^m \dot{h}^{li} \Big)$$

Quantum Optics Analogue:
Spontaneous Parametric Down-Conversion (SPDC)



produce squeezed entangled gravitons, of interest here

Evolution Operators → Towards evaluating Squeezing

Pure gravity (GW part)

$$\delta\mathcal{L}^{(2)}_{\mathrm{EH}} = -\frac{1}{4\kappa^2}\sqrt{-g}\left[\frac{1}{2}\nabla_\gamma h_{\alpha\beta}\nabla^\gamma h^{\alpha\beta} + R^\alpha{}_{\lambda\beta\gamma}h^\gamma_\alpha h^{\lambda\beta}\right].$$

$$S_{\mathrm{GW}} = \frac{1}{4}\sum_{\lambda=L,R}\int dt\int d^3\vec{k}\left(|\dot{h}_{\lambda,\vec{k}}|^2 - k^2|h_{\lambda,\vec{k}}|^2\right)$$

Evolution Operators → Towards evaluating Squeezing

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$$\hat{h}_{ij}(t,\vec{x}) = \int\frac{d^3\vec{k}}{(2\pi)^{3/2}}\frac{1}{\sqrt{2\Omega_k}}\sum_{\lambda=L,R}\left[e^{(\lambda)}_{ij,\vec{k}}\hat{\alpha}^\dagger_{\lambda,\vec{k}}e^{-ik\cdot x} + \text{H.c.}\right], \qquad [\hat{\alpha}_{\lambda,\vec{k}},\hat{\alpha}^\dagger_{\lambda',\vec{k}'}] = \delta_{\lambda,\lambda'}\delta^{(3)}(\vec{k}-\vec{k}')$$

$\Omega_k \equiv k$

Frequency of GW with momentum $\vec{k}$.

$$\boxed{\hat{\mathcal{H}}^{(0)}_{\rm GW} = \int d^3\vec{k}\,\Omega_k\sum_{\lambda=L,R}\hat{\alpha}^\dagger_{\lambda,\vec{k}}\hat{\alpha}_{\lambda,\vec{k}}}$$

$$\boxed{\mathcal{H}_I^{(1)}(t) = -\frac{\kappa}{2}\int d^3\vec{x}T^{ij}(t,\vec{x})h_{ij}(t,\vec{x}).}$$

Fourier $T^{ij}(t,-\vec{k}) = \int d^3\vec{x}e^{-i\vec{k}\cdot\vec{x}}T^{ij}(t,\vec{x})$

$$\hat{\mathcal{H}}_I^{(1)}(t) = -\sum_{\vec{k},\lambda}\left[\beta_\lambda(t,\vec{k})\hat{\alpha}^\dagger_{\lambda,\vec{k}} + \beta^\star_\lambda(t,\vec{k})\hat{\alpha}_{\lambda,\vec{k}}\right]$$

$$\beta_\lambda(t,\vec{k}) = \frac{1}{2}f_k e^{i\Omega_k t}e^{(\lambda)}_{ij}(\vec{k})T^{ij}(t,-\vec{k}),$$

Scattering matrix $\hat{S}^{(1)} = e^{-i\int_{-\infty}^{+\infty}dt\mathcal{H}^{(1)}_I(t)} = \exp\left[\sum_{\vec{k},\lambda}(\xi_{\lambda,\vec{k}}\hat{\alpha}^\dagger_{\lambda,\vec{k}} - \xi^\star_{\lambda,\vec{k}}\hat{\alpha}_{\lambda,\vec{k}})\right]$

$$= \prod_{\vec{k},\lambda}\hat{D}(\xi_{\lambda,\vec{k}})$$

$$\hat{D}(\xi_{\lambda,\vec{k}}) = \exp\left[\xi_{\lambda,\vec{k}}\hat{\alpha}^\dagger_{\lambda,\vec{k}} - \xi^\star_{\lambda,\vec{k}}\hat{\alpha}_{\lambda,\vec{k}}\right] \qquad \xi_{\lambda,\vec{k}} = i\int_{-\infty}^{+\infty}dt\beta_\lambda(t,\vec{k}).$$

GR axion-graviton terms : $\mathcal{L}_I^{(2)} = -\frac{\kappa^2}{2} h_{im} h^m{}_j \partial^i b \partial^j b$

$$\mathcal{H}_I^{(2)} = \frac{\kappa^2}{2} \int d^3\vec{x}\, h_{im} h^m_j \partial^i b \partial^j b,$$

$$= \frac{1}{2} \sum_{\vec{k},\vec{k}'} \sum_{\lambda,\lambda'} f_k f_{k'} \left(\int d^3\vec{x} \partial^i b \partial^j b e^{-i(\vec{k}+\vec{k}')\cdot\vec{x}} \right) \left(e_{im}^{(\lambda)}(\vec{k}) e_{mj}^{(\lambda')}(\vec{k}') \hat{\alpha}^\dagger_{\lambda,\vec{k}} \hat{\alpha}^\dagger_{\lambda',\vec{k}'} e^{i(\Omega_k+\Omega_{k'})\cdot t} \right) + \text{H.c.}$$

$$+ \frac{1}{2} \sum_{\vec{k},\vec{k}'} \sum_{\lambda,\lambda'} f_k f_{k'} \left(\int d^3\vec{x} \partial^i b \partial^j b e^{-i(\vec{k}+\vec{k}')\cdot\vec{x}} \right) \left(e_{im}^{(\lambda)}(\vec{k}) [e_{mj}^{(\lambda')}(\vec{k}')]^\dagger \hat{\alpha}^\dagger_{\lambda,\vec{k}} \hat{\alpha}_{\lambda',\vec{k}'} e^{i(\Omega_k-\Omega_{k'})\cdot t} \right) + \text{H.c.}$$

Gravitational Chern-Simons (gCS) terms:

$$\hat{\mathcal{H}}^{(2)}_{I,\mathrm{CS}} = \sum_{\vec{k},\vec{k}'}\sum_{\lambda,\lambda'} e^{i(\Omega_k+\Omega_{k'})\cdot t}\left(\int d^3\vec{x}\, b(x) e^{-i(\vec{k}+\vec{k}')\cdot\vec{x}}\right) f_{\lambda,\lambda'}(\vec{k},\vec{k}')\hat{a}^\dagger_{\vec{k},\lambda}\hat{a}^\dagger_{\vec{k}',\lambda'} + \mathrm{H.c.}$$
$$+ \sum_{\vec{k},\vec{k}'}\sum_{\lambda,\lambda'} e^{i(\Omega_k+\Omega_{k'})\cdot t}\left(\int d^3\vec{x}\, b(x) e^{-i(\vec{k}-\vec{k}')\cdot\vec{x}}\right) g_{\lambda,\lambda'}(\vec{k},\vec{k}')\hat{a}^\dagger_{\vec{k},\lambda}\hat{a}_{\vec{k}',\lambda'} + \mathrm{H.c.},$$

where

$$f_{\lambda,\lambda'}(\vec{k},\vec{k}') = iAf_k f_{k'}\Omega_k\Omega_{k'} l_{\vec{k}'} l_{\lambda'}\left(k'_m e^{(\lambda)}_{mj}(\vec{k}) k_l e^{(\lambda')}_{jl}(\vec{k}') - k_l k'_l e^{(\lambda)}_{mj}(\vec{k}) e^{(\lambda')}_{mj}(\vec{k}') + \Omega_k\Omega_{k'} e^{(\lambda)}_{jl}(\vec{k}) e^{(\lambda')}_{jl}(\vec{k}')\right),$$

$$g_{\lambda,\lambda'}(\vec{k},\vec{k}') = iAf_k f_{k'}\Omega_k\Omega_{k'} l_{\vec{k}'} l_{\lambda'}\left(-k'_m e^{(\lambda)}_{mj}(\vec{k}) k_l [e^{(\lambda')}_{jl}(\vec{k}')]^\dagger + k_l k'_l e^{(\lambda)}_{mj}(\vec{k})[e^{(\lambda')}_{mj}(\vec{k}')]^\dagger - \Omega_k\Omega_{k'} e^{(\lambda)}_{jl}(\vec{k})[e^{(\lambda')}_{jl}(\vec{k}')]^\dagger\right).$$

Squeezing

Squeezed State - quadratic interactions of gravitons

$$|\psi\rangle = \prod_{\lambda,\vec{k}} \hat{D}(\xi_{\lambda,\vec{k}})\hat{S}^{(2)}_{\mathrm{GR}}|0\rangle,$$

$$\hat{D}(\xi_{\lambda,\vec{k}}) = \exp\left[\xi_{\lambda,\vec{k}}\hat{\alpha}^{\dagger}_{\lambda,\vec{k}} - \xi^{\star}_{\lambda,\vec{k}}\hat{\alpha}_{\lambda,\vec{k}}\right],$$

$$\hat{S}^{(2)}_{\mathrm{GR}} = \exp\left[\frac{1}{2}\sum_{I,J}\mathcal{G}^{(\mathrm{GR})}_{IJ}\hat{\alpha}^{\dagger}_{I}\hat{\alpha}^{\dagger}_{J} - \mathrm{H.c.}\right],$$

multimode squeezing operator

GR axion-graviton terms (dominant over CS, *cf.* below):

$$\mathcal{H}_I^{(2)} = \frac{\kappa^2}{2}\int d^3\vec{x}\, h_{im} h_j^m \partial^i b \partial^j b,$$

$$= \frac{1}{2}\sum_{\vec{k},\vec{k}'}\sum_{\lambda,\lambda'} f_k f_{k'} \left(\int d^3\vec{x}\partial^i b \partial^j b e^{-i(\vec{k}+\vec{k}')\cdot\vec{x}}\right)\left(e_{im}^{(\lambda)}(\vec{k}) e_{mj}^{(\lambda')}(\vec{k}')\hat{\alpha}^\dagger_{\lambda,\vec{k}}\hat{\alpha}^\dagger_{\lambda',\vec{k}'} e^{i(\Omega_k+\Omega_{k'})\cdot t}\right) + \text{H.c.}$$

$$+ \frac{1}{2}\sum_{\vec{k},\vec{k}'}\sum_{\lambda,\lambda'} f_k f_{k'} \left(\int d^3\vec{x}\partial^i b \partial^j b e^{-i(\vec{k}+\vec{k}')\cdot\vec{x}}\right)\left(e_{im}^{(\lambda)}(\vec{k}) [e_{mj}^{(\lambda')}(\vec{k}')]^\dagger \hat{\alpha}^\dagger_{\lambda,\vec{k}}\hat{\alpha}_{\lambda',\vec{k}'} e^{i(\Omega_k-\Omega_{k'})\cdot t}\right) + \text{H.c.}$$

GR axion-graviton terms (dominant over CS, *cf.* below):

$$\mathcal{H}_I^{(2)} = \frac{\kappa^2}{2}\int d^3\vec{x}\, h_{im}h_j^m \partial^i b \partial^j b,$$

$$= \frac{1}{2}\sum_{\vec{k},\vec{k}'}\sum_{\lambda,\lambda'} f_k f_{k'}\left(\int d^3\vec{x}\partial^i b\partial^j b e^{-i(\vec{k}+\vec{k}')\cdot\vec{x}}\right)\left(e_{im}^{(\lambda)}(\vec{k})e_{mj}^{(\lambda')}(\vec{k}')\hat{\alpha}^\dagger_{\lambda,\vec{k}}\hat{\alpha}^\dagger_{\lambda',\vec{k}'}e^{i(\Omega_k+\Omega_{k'})\cdot t}\right) + \text{H.c.}$$

$$+\frac{1}{2}\sum_{\vec{k},\vec{k}'}\sum_{\lambda,\lambda'} f_k f_{k'}\left(\int d^3\vec{x}\partial^i b\partial^j b e^{-i(\vec{k}+\vec{k}')\cdot\vec{x}}\right)\left(e_{im}^{(\lambda)}(\vec{k})[e_{mj}^{(\lambda')}(\vec{k}')]^\dagger\hat{\alpha}^\dagger_{\lambda,\vec{k}}\hat{\alpha}_{\lambda',\vec{k}'}e^{i(\Omega_k-\Omega_{k'})\cdot t}\right) + \text{H.c.}$$

Superradiant condensate axion field in 2p axion state

$$b(t,r,\theta,\varphi) = \sum_{nlm} e^{-i\omega_{nlm}t}\sqrt{\frac{N_{nlm}}{2\mu_b}}\Psi_{nlm} + \text{c.c.},$$

Detweiler, PRD 22, 2323 (1980)
Britto, Cardoso, Pani, Lect. Notes Phys. 906, 1 (2015)

$$\boxed{\hat{\mathcal{H}}_I^{(2)} = \sum_{\vec{k},\vec{k}'}\sum_{\lambda,\lambda'} e^{i(\Omega_k+\Omega_{k'}-2\omega)\cdot t} f_k f_{k'} N_{2p}\left(\int d^3\vec{x}\frac{\partial^i\Psi_{2p}\partial^j\Psi_{2p}}{4\mu_b}\right. \times e^{-i(\vec{k}+\vec{k}')\cdot\vec{x}}e_{im}^{(\lambda)}(\vec{k})e_{mj}^{(\lambda')}(\vec{k}')\left.\hat{\alpha}^\dagger_{\lambda,\vec{k}}\hat{\alpha}^\dagger_{\lambda',\vec{k}'}\right) + \text{H.c.} \quad (6.1)}$$

N_{2p} = number of Axions in 2p state

Multimode Squeezed graviton States

Evolution operator has the form of multimode squeezing operator

$$\boxed{\hat{S} = \exp\left[\frac{1}{2}\sum_{I,J}\mathcal{G}_{IJ}\;\hat{\alpha}_I^\dagger\hat{\alpha}_J^\dagger - h.c.\right]}$$

$J, I = (\lambda, \vec{k})$

Graviton states

$$\boxed{\mathcal{G}_{IJ} = -2\;i\;\mathcal{F}_{IJ}\;T\;\mathrm{sinc}\left[(\Omega_k + \Omega_{k'} - E)\,\frac{T}{2}\right]}$$

***T* = life time of cloud**

$$T\mathrm{sinc}\left[(\Omega_k + \Omega_{k'} - E)\,(T/2)\right] \overset{T\to\infty}{=} \delta\left(\Omega_k + \Omega_{k'} - E\right)$$

Hence,

$$\hat{S}^\dagger\hat{\alpha}_I\hat{S} = \sum_J(\mu_{IJ}\hat{\alpha}_J + \nu_{IJ}\hat{\alpha}_J^\dagger)\;,\qquad \mu_{IJ} = \delta_{IJ} + \frac{1}{2!}\sum_M \mathcal{G}_{IM}\mathcal{G}^\star_{MJ}\;+\;\ldots\;,$$

$$\nu_{IJ} = \mathcal{G}_{IJ} + \frac{1}{3!}\sum_{M,L}\mathcal{G}_{IM}\mathcal{G}^\star_{ML}\mathcal{G}_{LJ}\;+\;\ldots\;.$$

Number of squeezed graviton states

$$\boxed{\langle N_{gr}\rangle = \sum_{I,J}\left|\nu_{IJ}\right|^2 \lesssim \sum_{I,J}\left(\left|\mathcal{G}_{IJ}\right|^2\;+\;\ldots\right)}$$

Some details: **2p axion state – GR terms** **Superradiance**

$$\mathcal{F}^{(GR)}_{(\vec{k},\lambda)(\vec{k}',\lambda')} = f_k f_{k'} N_{2p} \left(\int d^3\vec{x}\, \frac{\partial^i \Psi_{2p}\, \partial^j \Psi_{2p}}{4\mu_b}\, e^{-i(\vec{k}+\vec{k}')\cdot\vec{x}}\, e^{(\lambda)}_{im}(\vec{k})\, e^{(\lambda')}_{mj}(\vec{k}') \right)$$

N_{2p} = number of Axions in 2p state

enhancement factor $N_{2p} >> 1$ axion states

For long cloud lifetimes *T*

$$\mathcal{G}^{(GR)}_{IJ} \approx -2i\, T \mathcal{F}^{(GR)}_{IJ} \qquad \mathcal{F}^{(GR)}_{(\vec{k},\lambda)(\vec{k}',\lambda')} = \frac{f_k f_{k'} N_{2p}}{4\mu_b} e^{(\lambda)}_{im}(\vec{k})\, I_{ij}(\vec{k},\vec{k}')\, e^{(\lambda')}_{mj}(\vec{k}')$$

$$I_{ij}(\vec{k},\vec{k}') = \int d^3\vec{x}\, (\partial_i \Psi_{2p}(\vec{x}))\, (\partial_j \Psi_{2p}(\vec{x}))\, e^{-i(\vec{k}+\vec{k}')\cdot\vec{x}}$$

$$\Psi_{2p}(\vec{x}) = \frac{1}{8\sqrt{\pi}} \frac{r}{r_0^{5/2}} e^{-\frac{r}{2r_0}}\, e^{i\varphi} \sin\theta\,, \qquad r_0 = (a_\mu \mu_b)^{-1}$$

Some details:

2p axion state – CS terms

Superradiance

For long cloud lifetimes *T*

enhancement factor
N_{2p} >> 1 axion states

$$\mathcal{G}^{(CS)}_{IJ} \approx -2i\, T \mathcal{F}^{(CS)}_{IJ} \qquad \mathcal{F}^{(CS)}_{(\lambda,\vec{k})(\lambda',\vec{k}')} = iA\sqrt{\frac{N_{2p}}{2\mu_b}} f_k f_{k'}\ \Omega_k^2\ \Omega_{k'}^2\ \mathcal{I}^{(CS)}_{(\lambda,\vec{k})(\lambda',\vec{k}')}$$

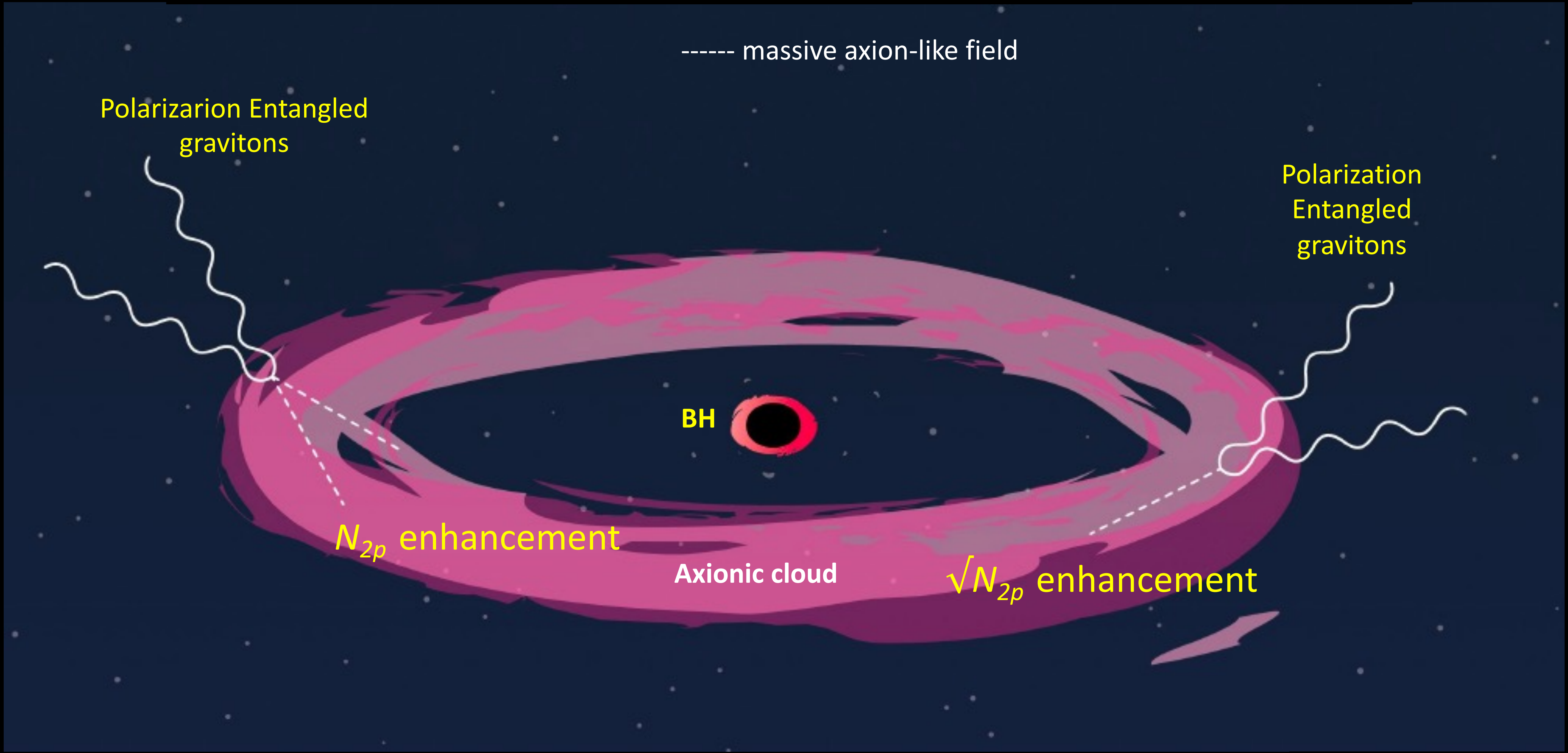
$$\mathcal{I}^{(CS)}_{(\lambda,\vec{k})(\lambda',\vec{k}')} = l_{\vec{k}'} l_{\lambda'} \widetilde{\Psi}_{2p}(\vec{k}+\vec{k}') \left([e^{(3)}(\vec{k}')]_m\ e^{(\lambda)}_{mj}(\vec{k})\ e^{(\lambda')}_{jl}(\vec{k}')[e^{(3)}(\vec{k})]_l + (1-\cos\Delta\theta)\ e^{(\lambda)}_{mj}(\vec{k})\ e^{(\lambda')}_{mj}(\vec{k}') \right)$$

Fourier transform of $\Psi_{2p}(\vec{x})$

$$\widetilde{\Psi}_{2p}(\vec{q}) = \frac{128 i\sqrt{\pi}\ q\ r_0^{5/2}}{(4q^2 r_0^2+1)^3} \sin\theta_q e^{i\varphi_q} \qquad r_0 = (a_\mu \mu_b)^{-1}$$

$$\vec{q} = -(\vec{k}+\vec{k}').$$

Different enhancement factors between GR and CS terms



CS squeezing is subleading to GR-induced one by many orders of magnitude

Courtesy N Kouniniotis

Important technical remarks on the above estimates of the Graviton squeezing Evolution operator

Mathematical Comparison: Dyson Series vs. Magnus Expansion

Quantum Mechanical Evolution operator *U(t)*:

$$\frac{d}{dt}U(t) = A(t)U(t), \quad U(0) = I$$

Feature	**Dyson Series**	**Magnus Expansion**
Mathematical Form	Expands the time-evolution operator $U(t)$ as an infinite sum of time-ordered integrals.	Expands the logarithm of the evolution operator as a sum of integrals involving nested commutators ($U(t) = e^{\Omega(t)}$).
Time-Ordering	Requires the explicit use of a time-ordering operator $\mathcal{T}$ for all terms.	Eliminates the time-ordering operator; instead, time information is implicitly contained within nested commutators.
Physical Properties	Truncated series generally loses strict unitarity (or probability conservation).	Truncated series perfectly preserves qualitative physical properties like unitarity and symplectic structure.
Simplicity	Formal power series are much simpler and straightforward to derive at higher orders.	Expressions become increasingly intricate at higher orders due to the use of Bernoulli numbers and complex commutators.
Common Use Cases	Analytical quantum field theory, Feynman diagrams, and linear perturbation theory.	Geometric numerical integration, classical mechanics, and control engineering in quantum computing.

Table 1: Comparison between Dyson Series and Magnus Expansion.

Mathematical Comparison: Dyson Series vs. Magnus Expansion

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Table 1: Comparison between Dyson Series and Magnus Expansion.

More suitable to us, upon the assumption that graviton emission is a ***rare event***
→ ***Hamiltonians @ spacelike separations commute***

Dyson vs Magnus expansion of Evolution Operators

The Dyson Series Formula

$$U(t) = I + \sum_{n=1}^{\infty} U_n(t)$$

- **First Order (U_1):**

$$U_1(t) = \int_0^t A(t_1)\, dt_1$$

- **Second Order (U_2):**

$$U_2(t) = \int_0^t \int_0^{t_1} A(t_1)A(t_2)\, dt_2\, dt_1$$

- **General n-th Order (U_n):**

$$U_n(t) = \int_0^t \int_0^{t_1} \cdots \int_0^{t_{n-1}} A(t_1)A(t_2)\ldots A(t_n)\, dt_n \ldots dt_2\, dt_1$$

Compact Time-Ordered Form

$\mathcal{T}$ rearranges operators chronologically from right to left

$$U(t) = \mathcal{T}\left\{\exp\left(\int_0^t A(t')\, dt'\right)\right\}$$

Dyson vs Magnus expansion of Evolution Operators

The Magnus Expansion Formula

expresses the evolution operator as a single exponential:

$$U(t) = \exp(\Omega(t)) \quad \text{where} \quad \Omega(t) = \sum_{k=1}^{\infty} \Omega_k(t)$$

- **First Order (Ω_1):**

$$\Omega_1(t) = \int_0^t A(t_1)\, dt_1$$

- **Second Order (Ω_2):**

$$\Omega_2(t) = \frac{1}{2} \int_0^t \int_0^{t_1} [A(t_1), A(t_2)]\, dt_2\, dt_1$$

- **Third Order (Ω_3):**

$$\Omega_3(t) = \frac{1}{6} \int_0^t \int_0^{t_1} \int_0^{t_2} \left([A(t_1), [A(t_2), A(t_3)]] + [A(t_3), [A(t_2), A(t_1)]] \right) dt_3\, dt_2\, dt_1$$

General Recursive Formula

$$\Omega'(t) = \sum_{j=0}^{\infty} \frac{B_j}{j!} \mathrm{ad}^j_{\Omega(t)}(A(t))$$

In our case of superradiant axion cloud studies : Assume rare event rates → omit time ordering in Dyson;s formula
Cleare seen in Magnus expansion , which is an expansion of the exponent according to the commutators of the Hamiltonian at different times. Specifically, the Magnus expansion

$$\hat{S} = \exp\left[\sum_n M_n\right],$$

M_n related to commutators of the Hamiltonian at different times. Indeed, in Dyson's expansion:

$$\hat{S} = I + \sum_n P_n.$$

Then, there are the following relations between the two expansions

$$M_1 = P_1,$$

$$M_2 = P_2 - P_1^2,$$

$$M_3 = P_3 - \frac{1}{2}(P_1 P_2 + P_2 P_1) + \frac{1}{3} P_1^3,$$

$$M_4 = \ldots,$$

In our case of graviton production (similarly in the cases of SPDC and SFWM in Quantum Optics) we assume that it obeys

$$[\mathcal{H}_{\rm int}(x'), \mathcal{H}_{\rm int}(x)] = 0$$

i.e. emissions **are rare**, occurring at **space-like separated** spacetime events

)

Graviton-Squeezing GR terms

$$\frac{d^2}{d\Omega d\Omega'}\sum_{I,J}\left|\left(\mathcal{G}^{(GR)}_{IJ}\right)\right|^2 = \frac{1}{128\pi^3}\left(\frac{\mu_b}{M_{\rm Pl}}\right)^4 N_{2p}^2\, T\mu_b$$

$$\times \sum_{\lambda,\lambda'}\int d\tilde{k}d\tilde{k}'\ \tilde{k}\tilde{k}'\delta\left(\tilde{k}+\tilde{k}'-2\right)\left|\widetilde{\mathcal{I}}^{(GR)}_{(\lambda,\vec{k})(\lambda',\vec{k}')}\right|^2 ,$$

$$\tilde{k}^{(\prime)} = k^{(\prime)}/\mu_b \text{ and } \mathcal{I} = \widetilde{\mathcal{I}}/\mu_b^2$$

Axion mass: μ_b

Upper bound: almost collinear gravitons

$$\boxed{\mathcal{O}(0.1) \leq \sum_{I,J}|\mathcal{G}^{(GR)}_{IJ}|^2 \lesssim 2.5\times 10^{-15}\, T\mu_b\,.}$$

NB: Entangled 2-graviton state

$$\boxed{|\Psi_{GR}\rangle = \frac{1}{2}\left(\mathcal{G}^{(GR)}_{(R,\vec{k})(L,\vec{k}')}|RL\rangle + \mathcal{G}^{(GR)}_{(L,\vec{k})(R,\vec{k}')}|LR\rangle + \mathcal{G}^{(GR)}_{(L,\vec{k})(L,\vec{k}')}|LL\rangle + \mathcal{G}^{(GR)}_{(R,\vec{k})(R,\vec{k}')}|RR\rangle\right)}$$

with $\mathcal{G}^{(GR)}_{(R,\vec{k})(L,\vec{k}')}, \mathcal{G}^{(GR)}_{(L,\vec{k})(R,\vec{k}')} \gg \mathcal{G}^{(GR)}_{(L,\vec{k})(L,\vec{k}')}, \mathcal{G}^{(GR)}_{(R,\vec{k})(R,\vec{k}')}$

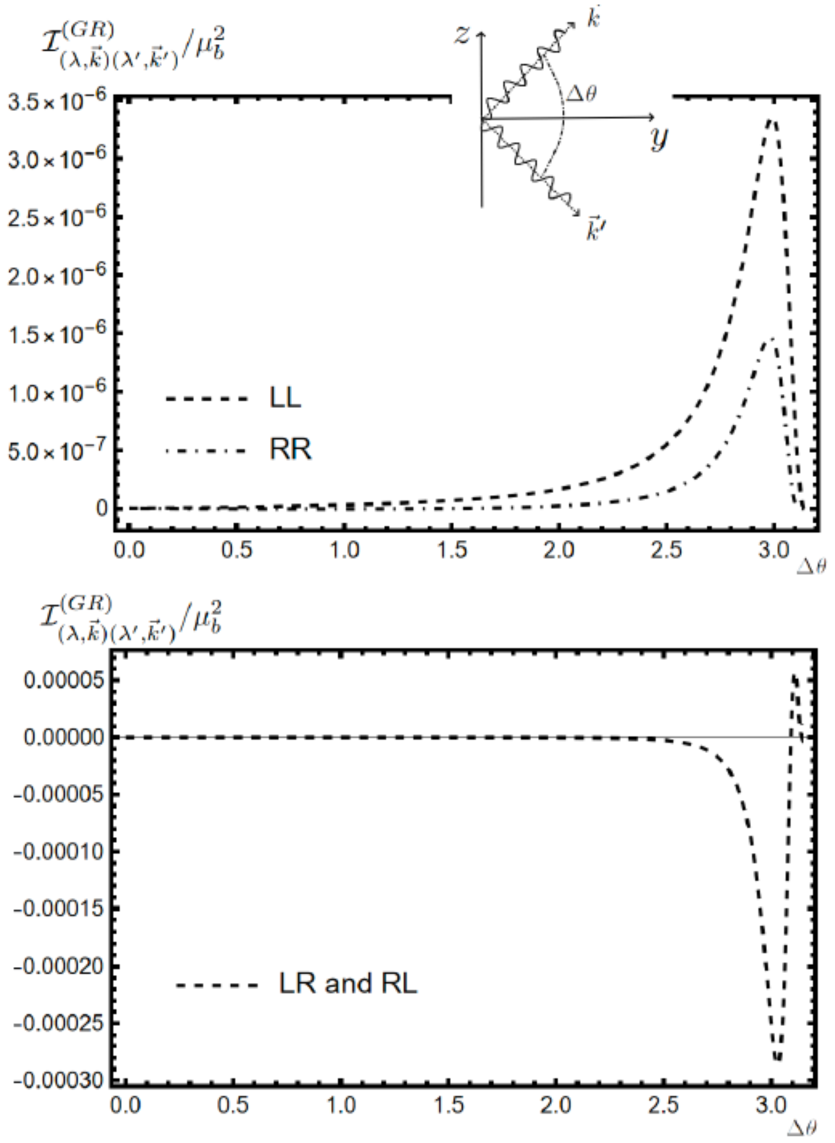

FIG. 2. Angular and polarization correlations for the GR interaction. The $2p$-state results in the asymmetry between the LL and RR pairs. The plot corresponds to $a_\mu = 0.1$.

Graviton-Squeezing GR terms

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$$\times\sum_{\lambda,\lambda'}\int d\tilde{k}d\tilde{k}'\;\tilde{k}\tilde{k}'\delta\left(\tilde{k}+\tilde{k}'-2\right)\left|\widetilde{\mathcal{I}}^{(GR)}_{(\lambda,\vec{k})(\lambda',\vec{k}')}\right|^2 ,$$

$\tilde{k}^{(\prime)} = k^{(\prime)}/\mu_b$ and $\mathcal{I} = \widetilde{\mathcal{I}}/\mu_b^2$ **Axion mass: μ_b**

$$\sum_{I,J}|\mathcal{G}_{IJ}^{(GR)}|^2 \lesssim 2.5\times 10^{-15}\, T\mu_b$$

$$\tau_s = \frac{1}{\omega_I(2p)} = 24\,\mu_b^{-1}\left(\frac{\alpha}{G\mathcal{M}}\right)^{-1} a_\mu^{-8}$$

Life time of cloud can be sufficiently long compared to the characteristic scale τ_s superradiance is effective,
eg $T > 10^7\,\tau_s$ → significant multimode squeezing

$\alpha/(G\mathcal{M}) \sim \mathcal{O}(1)$

$\mathcal{M}$ = **Rotating Black Hole Mass**

$a_\mu \equiv G\mathcal{M}\mu_b = 0.1$

$$\mathcal{O}(0.1) \leq \sum_{I,J}|\mathcal{G}_{IJ}^{(GR)}|^2 \lesssim \mathcal{O}(60-75)$$

$$\langle N_{gr}\rangle \gg 1$$

Graviton-Squeezing CS terms

e.g. Stringy RVM Cosmology, Basilakos, NEM, Solà Peracaula

String-inspired models for string-model independent axion (assuming it gets a mass μ_b via some mechanism)

$$S_{CS} = -\frac{A}{4}\int d^4x\sqrt{-g}\;\; b\;\; R_{\mu\nu\rho\sigma}\widetilde{R}^{\nu\mu\rho\sigma}$$

$$A \sim 10^{-2} M_{\rm Pl}/M_s^2$$

$$\boxed{\sum_{I,J}\left|(\mathcal{G}_{IJ}^{(CS)})\right|^2 \lesssim 10^{-10}\left(\frac{\mu_b}{M_s}\right)^4 \mu_b T}$$

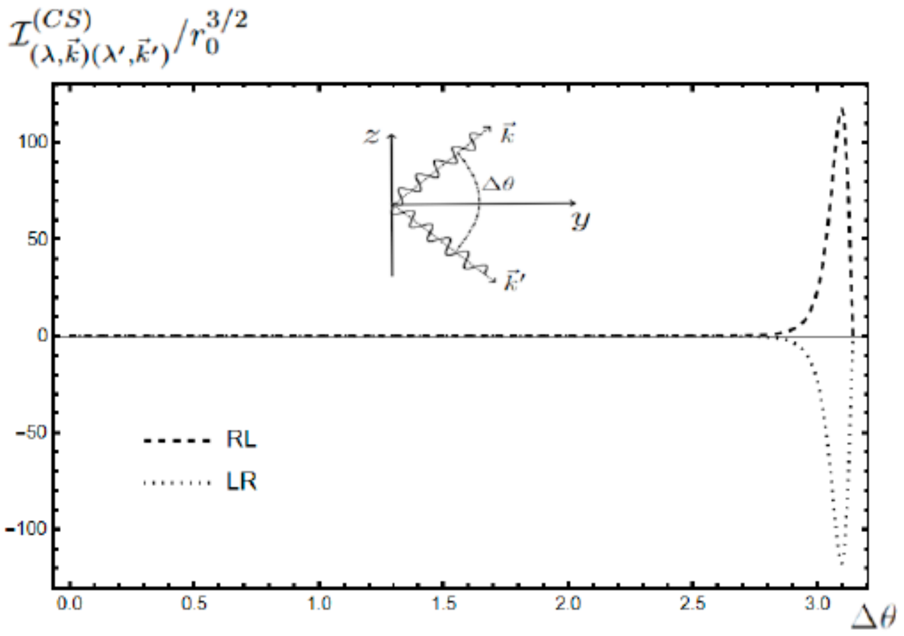

FIG. 3. Angular and polarization correlations for the CS interaction. Only pairs of opposite polarizations are produced; Maximal entanglement occurs between the L and R polarizations. The plot corresponds to $a_\mu = 0.1$.

NB: Maximally entangled 2-graviton state (Bell type)

$$\boxed{|\Psi_{CS}\rangle = \frac{1}{2}\mathcal{G}^{(CS)}_{(R,\vec{k})(L,\vec{k}')}\left(|LR\rangle - |RL\rangle\right)}$$

since $\mathcal{G}^{(CS)}_{(R,\vec{k})(L,\vec{k}')} = -\mathcal{G}^{(CS)}_{(L,\vec{k})(R,\vec{k}')}$

Graviton-Squeezing CS terms

e.g. high M_s , Stringy RVM inflationary Cosmology, Basilakos, Dorlis, NEM, Solà Peracaula, Vlachos

String-inspired models for string-model independent axion (assuming it gets a mass μ_b via some mechanism)

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$$\boxed{\sum_{I,J}\left|\left(\mathcal{G}_{IJ}^{(CS)}\right)\right|^2 \lesssim 10^{-10}\left(\frac{\mu_b}{M_s}\right)^4 \mu_b T}$$

Significant suppression factor , especially for high string scales M_s

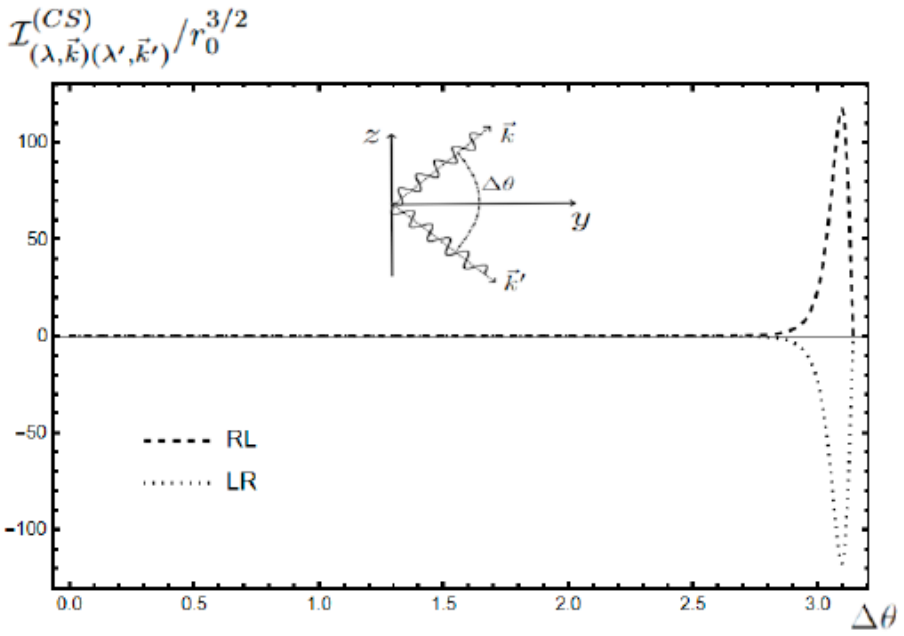


FIG. 3. Angular and polarization correlations for the CS interaction. Only pairs of opposite polarizations are produced; Maximal entanglement occurs between the L and R polarizations. The plot corresponds to $a_\mu = 0.1$.

Hence, even for long-lived clouds CS-induced graviton squeezing will be suppressed compared to GR-induced one

$$\boxed{\langle N_{gr}\rangle^{\rm CS} << \langle N_{gr}\rangle^{\rm GR}}$$

Outlook

What can we do with these results?

❖ Compute the polarization entangled graviton correlators and study their symmetries

❖ Do axion phenomenology → by not having observed graviton squeezing so far at current interferometers
→ impose upper bounds on the life times of the axion clouds

❖ Hope for observations of the polarization-entangled squeezed states in the future

Ευχαριστώ Πολύ !

Thank you !

SPARES

Why a Linear-Axion potential leads to Inflation?

Dorlis, Vlachos, NEM
PRD110 (2024), 063512
2403.09005 [gr-qc]

Linear axion potential and Running Vacuum Model (RVM) Inflation

Dynamical System approach to inflation

$$3H^2 = \kappa^2\left(\frac{\dot{b}^2}{2} + V(b)\right)$$

$$2\dot{H} + 3H^2 = -\kappa^2\left(\frac{\dot{b}^2}{2} - V(b)\right)$$

$$\ddot{b} + 3H\dot{b} + V_{,b} = 0$$

$$x' = -\frac{3}{2}\left[2x - x^3 + x\left(y^2 - 1\right) - \frac{\sqrt{2}}{\sqrt{3}}\lambda y^2\right]$$

$$y' = -\frac{3}{2}y\left[-x^2 + y^2 - 1 + \frac{\sqrt{2}}{\sqrt{3}}\lambda x\right]$$

$$\lambda' = -\sqrt{6}\left(\Gamma - 1\right)\lambda^2 x$$

$$\lambda = -\frac{V_{,b}}{\kappa V} \quad \text{and} \quad \Gamma = \frac{V V_{,bb}}{V_{,b}^2}$$

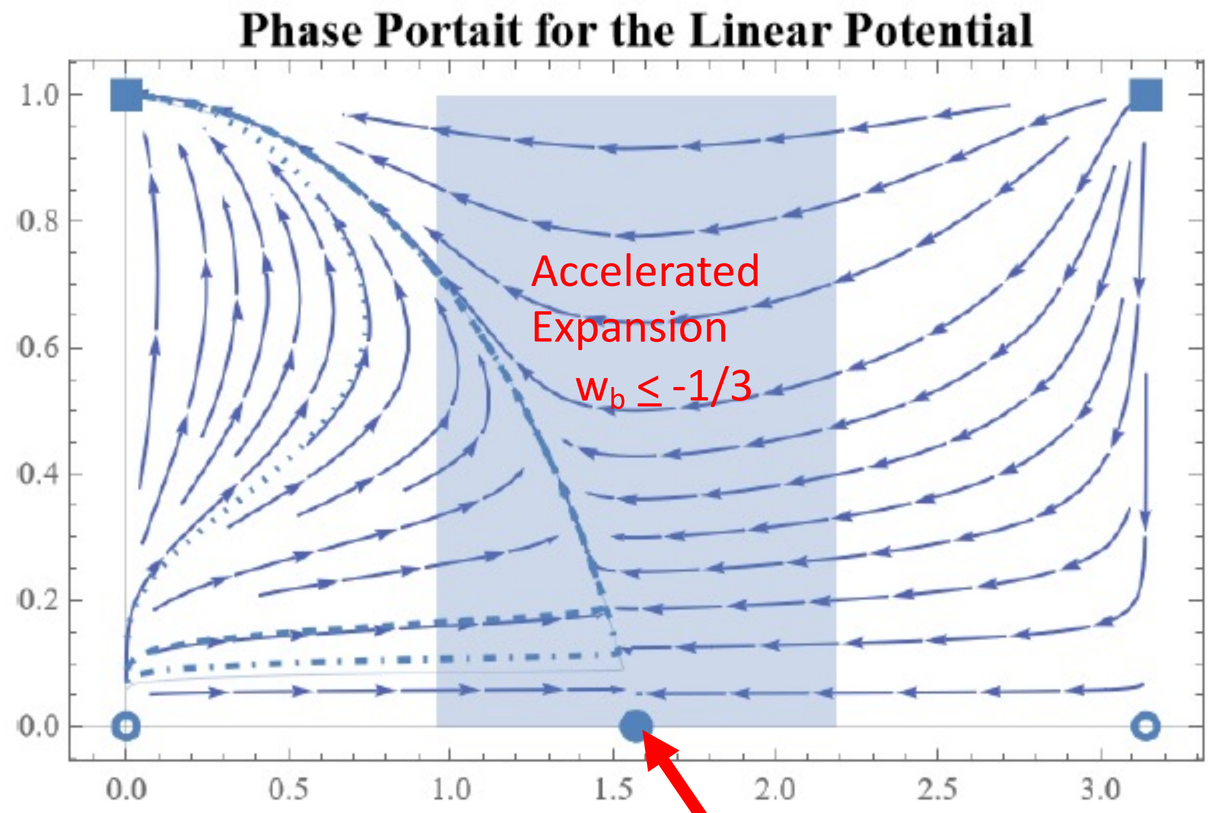
$$x = cos\varphi \;,\; y = sin\varphi$$

$$\boxed{\begin{aligned}\varphi' &= \left(3\cos\varphi - \frac{\sqrt{6}}{2}\lambda\right)\sin\varphi \\ \lambda' &= -\sqrt{6}(\Gamma - 1)\lambda^2\cos\varphi\end{aligned}}$$

$$\zeta = \frac{\lambda}{\lambda + 1}$$

$$\varphi' = \left(3\cos\varphi - \frac{\sqrt{6}}{2}\frac{\zeta}{1-\zeta}\right)\sin\varphi$$

$$\zeta' = -\sqrt{6}(\Gamma - 1)\zeta^2\cos\varphi$$

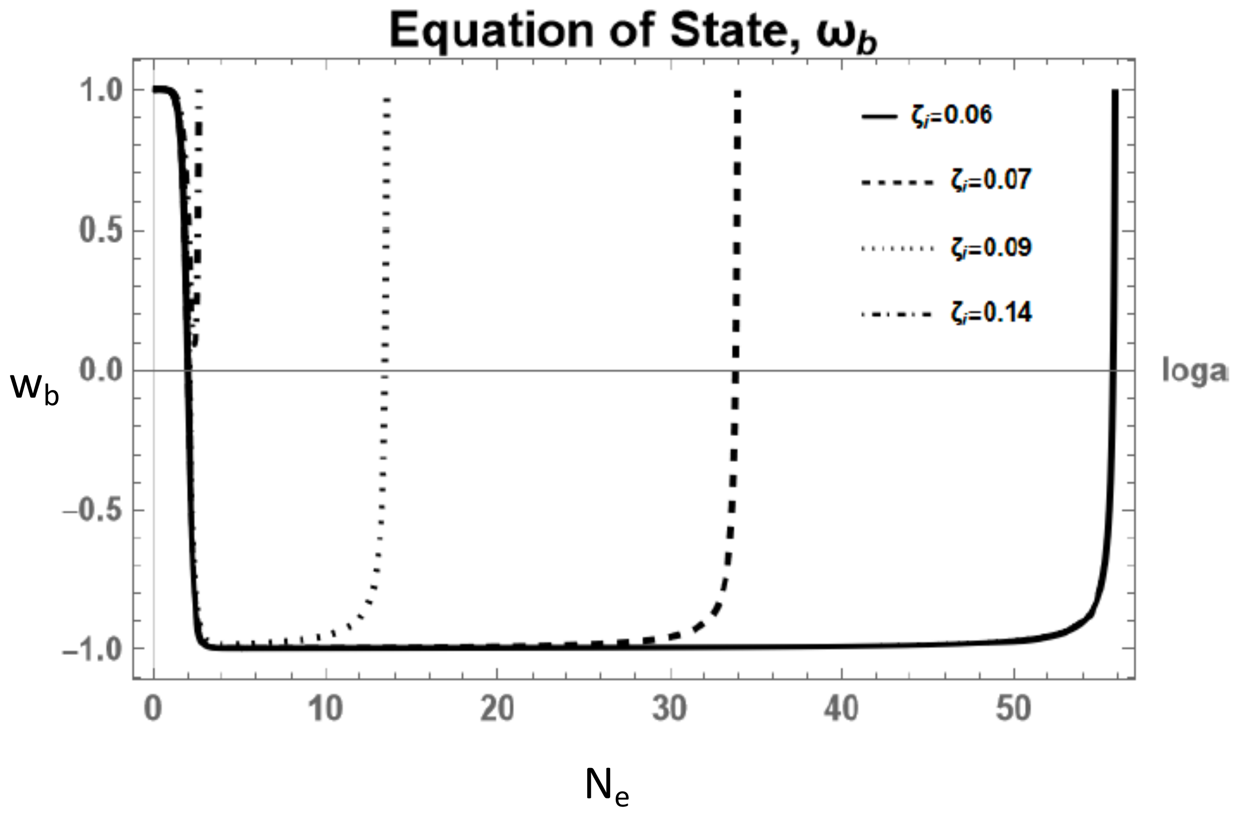


Inflation (exponential Expansion $w_b = -1$)

$\zeta - \varphi$ plane.

$$\zeta = \frac{\lambda}{\lambda + 1} \qquad \varphi' = \left(3\cos\varphi - \frac{\sqrt{6}}{2}\frac{\zeta}{1-\zeta}\right)\sin\varphi$$
$$\zeta' = -\sqrt{6}(\Gamma - 1)\zeta^2 \cos\varphi$$

The condensate $\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle$ induced by **quantum graviton** fluctuations of chiral (left-right asymmetric) GW type



$$\langle R_{CS}\rangle^{total} = \mathcal{N}_I \frac{A\kappa^4\mu^4}{\pi^2}\dot{b}_I H_I^3$$

$$\boxed{R_{CS} = \frac{1}{2}R^{\mu}{}_{\nu\rho\sigma}\tilde{R}^{\nu}{}_{\mu}{}^{\rho\sigma}}$$

$$A = \sqrt{\frac{2}{3}}\frac{\alpha'}{48\kappa}$$

String EFT:
$\mu = M_s = (\alpha')^{-1/2}$

Evolution of equation of state for the orbits of the linear b-potential phase space

For some initial value of φ_i inflation with e-foldings $N_e > 50$ is achieved for $\zeta < 0.06$ (**inflation → saddle point**)

$$\boxed{\zeta = \frac{\lambda}{\lambda+1} \qquad \varphi' = \left(3\cos\varphi - \frac{\sqrt{6}}{2}\frac{\zeta}{1-\zeta}\right)\sin\varphi \qquad \zeta' = -\sqrt{6}(\Gamma - 1)\zeta^2\cos\varphi}$$

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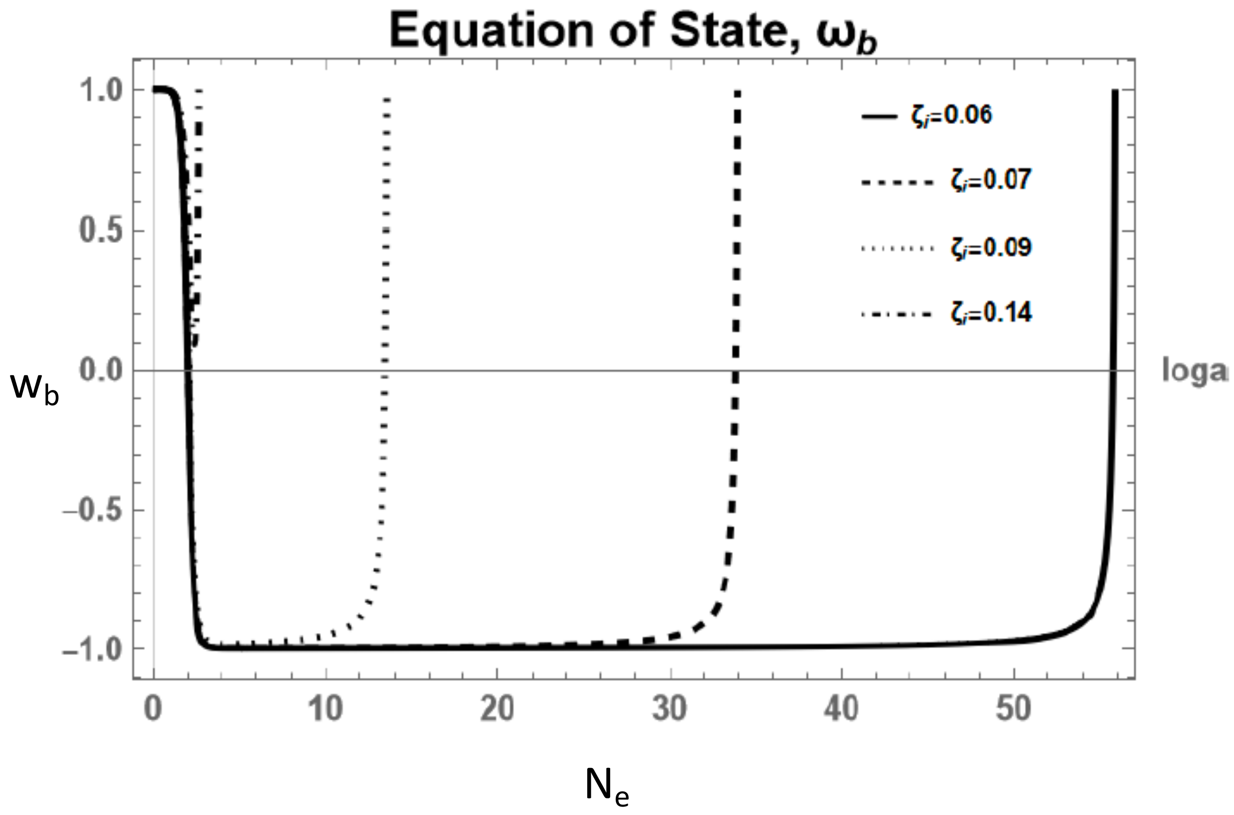

$$\langle R_{CS}\rangle^{total} = \mathcal{N}_I \frac{A\kappa^4\mu^4}{\pi^2}\dot{b}_I H_I^3$$

Hubble

GW sources

UV cutoff of graviton modes

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$$\boxed{\dot{b}_I \sim 10^{-1} H_I M_{\rm Pl}}$$

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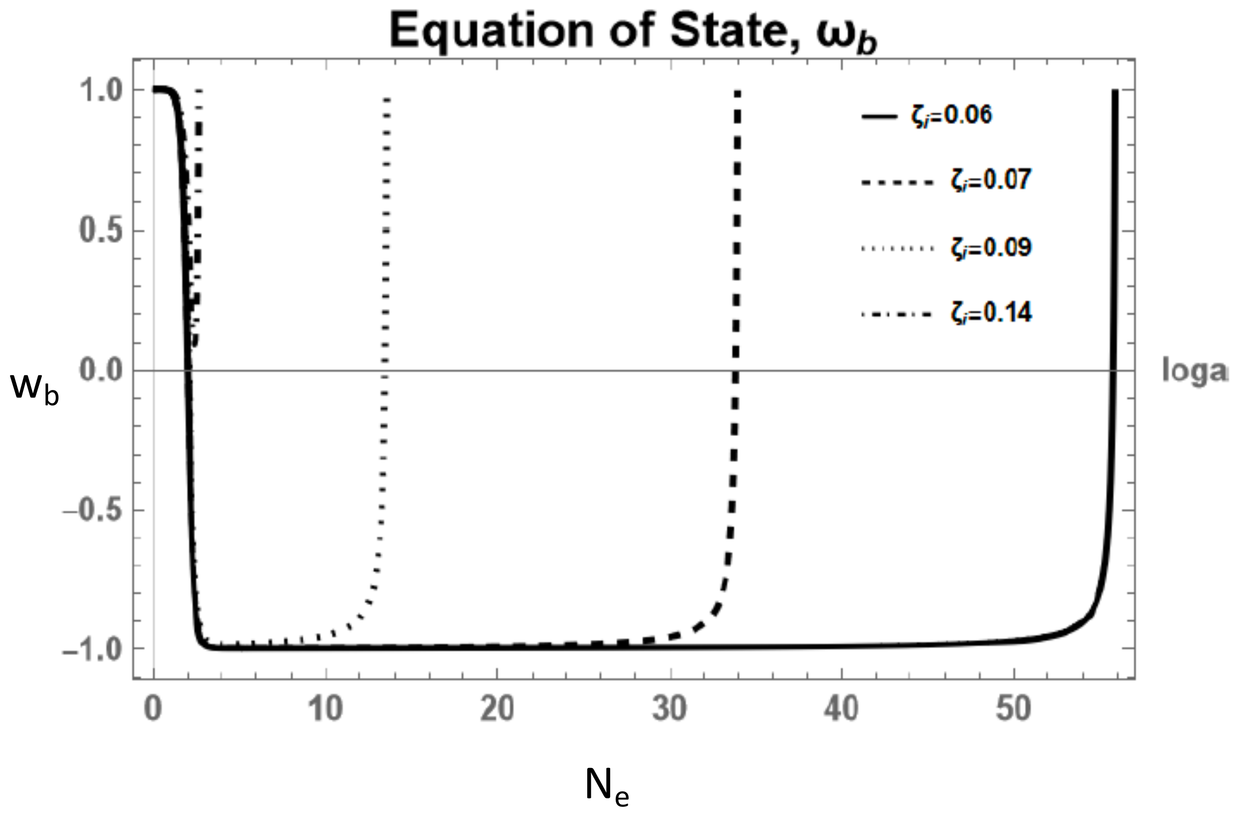

The condensate $\langle R_{\mu\nu\rho\sigma}\widetilde{R}^{\mu\nu\rho\sigma}\rangle$ induced

by **quantum gr** ... ral

(left-right

Treating **CS Gravity as EFT**

-**Canonical Quantization** of

graviton modes

$$\langle R_{CS}\rangle^{total} = \mathcal{N}_I \frac{A\kappa^4\mu^4}{\pi^2}\dot{b}_I H_I^3$$

GW sources

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UV cutoff of graviton modes

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The Graviton Correlators and Entangled States

NB:

GR correlations in absence of anomalies

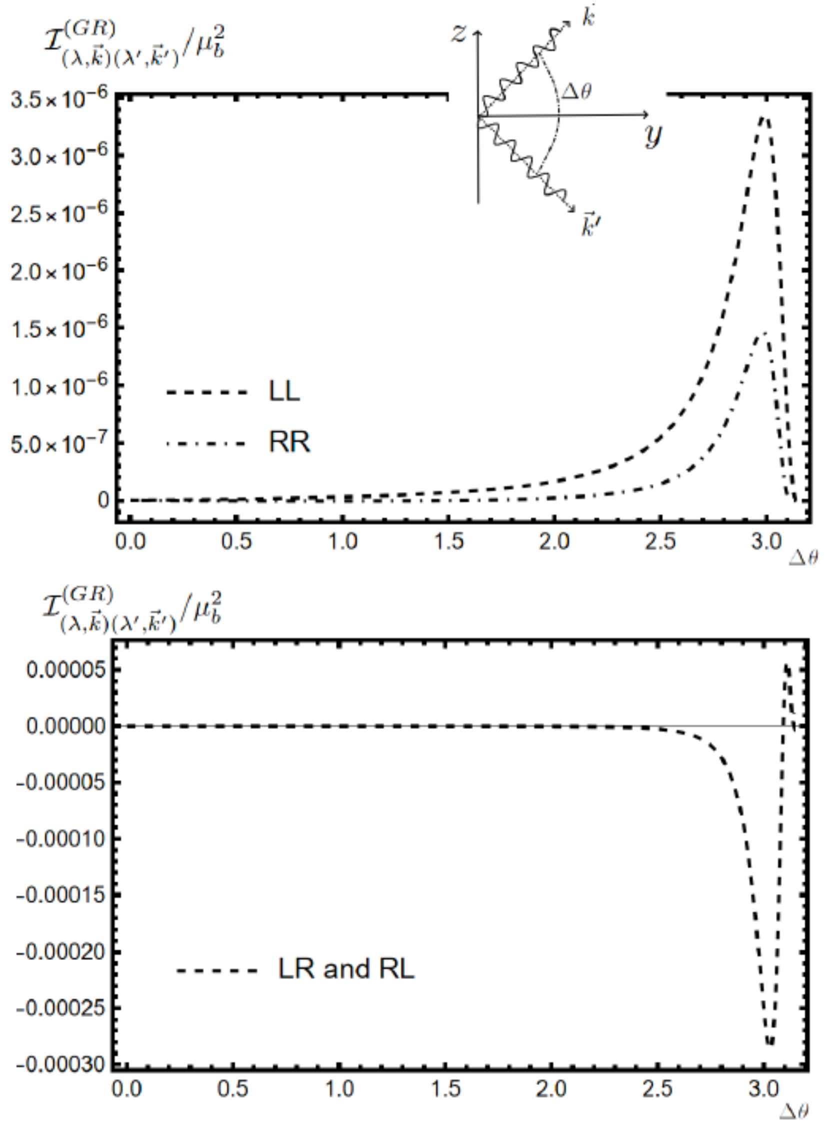
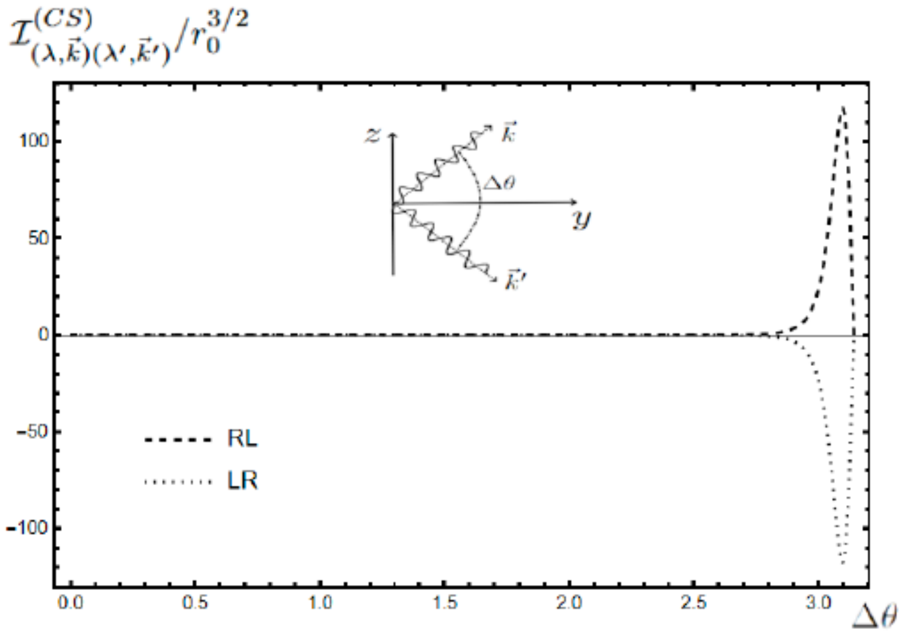


FIG. 2. Angular and polarization correlations for the GR interaction. The 2p-state results in the asymmetry between the LL and RR pairs. The plot corresponds to $a_\mu = 0.1$.

Contributions of CS gravitational Anomaly in EPR entangled graviton correlations



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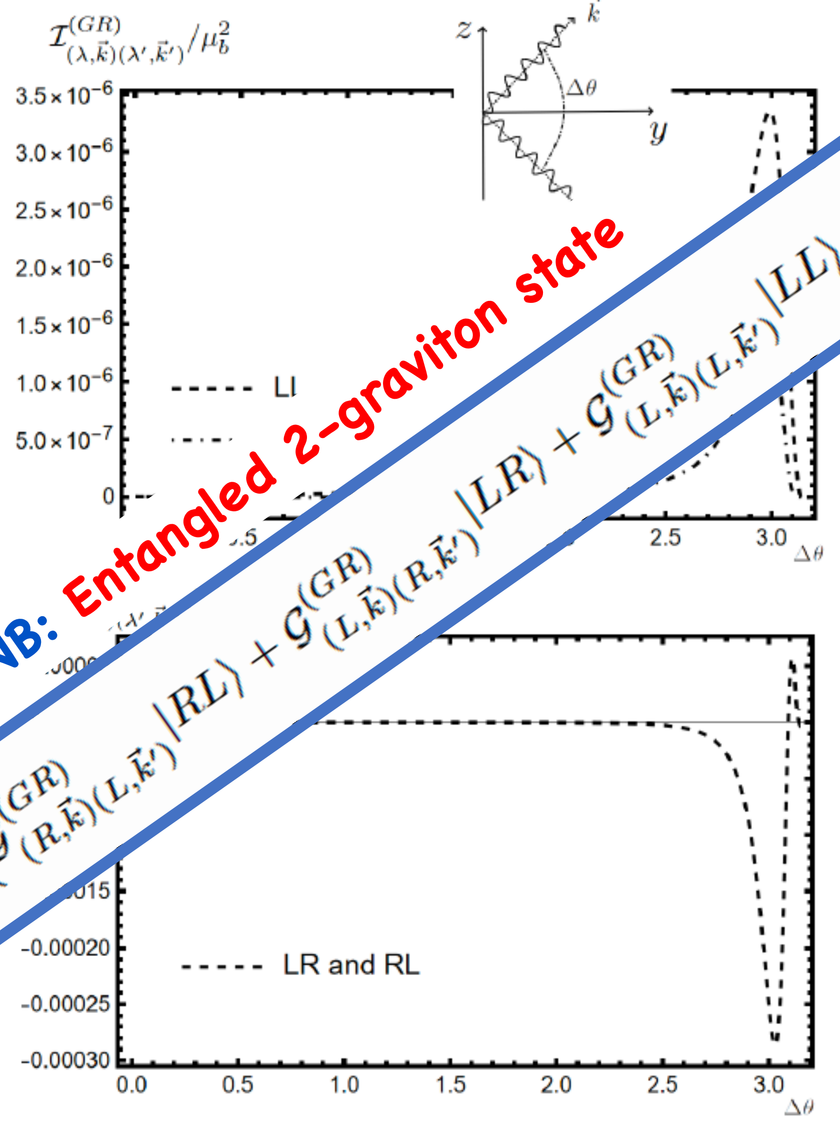

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NB: Entangled 2-graviton state

$$|\Psi_{GR}\rangle = \frac{1}{2}\left(\mathcal{G}^{(GR)}_{(R,\vec k)(L,\vec k')}|RL\rangle + \mathcal{G}^{(GR)}_{(L,\vec k)(R,\vec k')}|LR\rangle + \mathcal{G}^{(GR)}_{(L,\vec k)(L,\vec k')}|LL\rangle + \mathcal{G}^{(GR)}_{(R,\vec k)(R,\vec k')}|RR\rangle\right)$$

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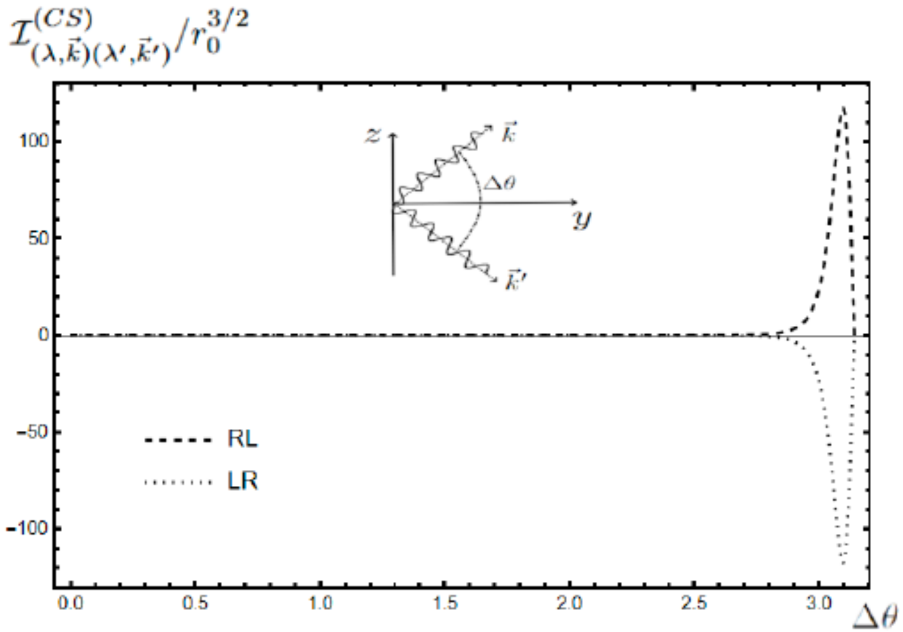

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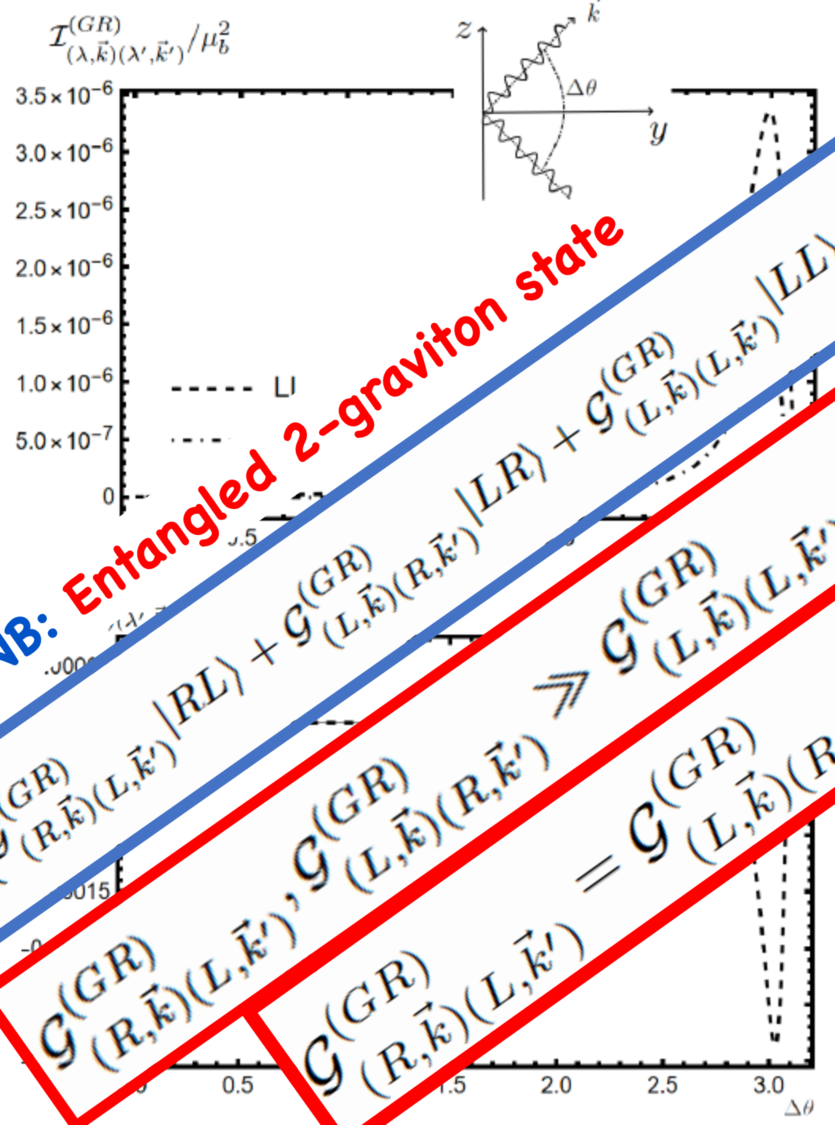

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with

$$\mathcal{G}^{(GR)}_{(R,\vec{k})(L,\vec{k}')}, \mathcal{G}^{(GR)}_{(L,\vec{k})(R,\vec{k}')} \gg \mathcal{G}^{(GR)}_{(L,\vec{k})(L,\vec{k}')}, \mathcal{G}^{(GR)}_{(R,\vec{k})(R,\vec{k}')}$$

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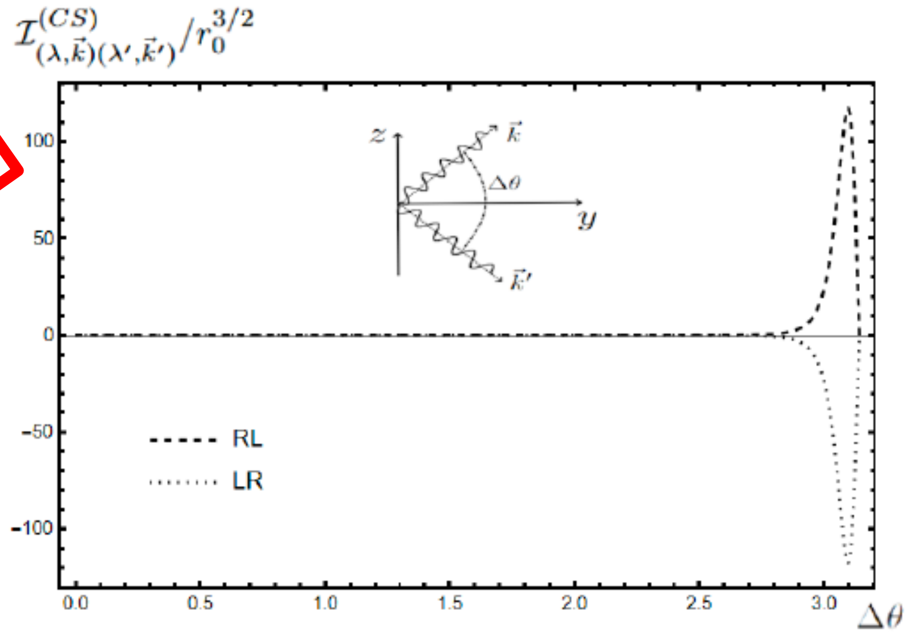

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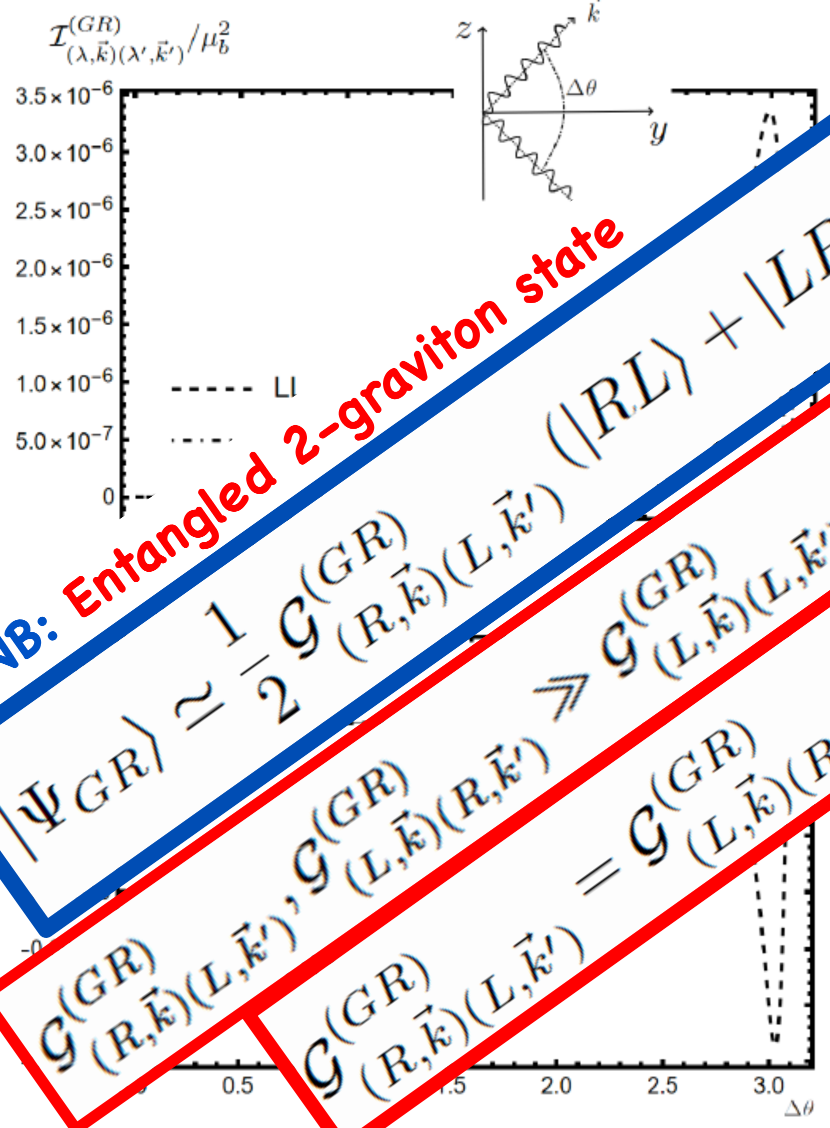

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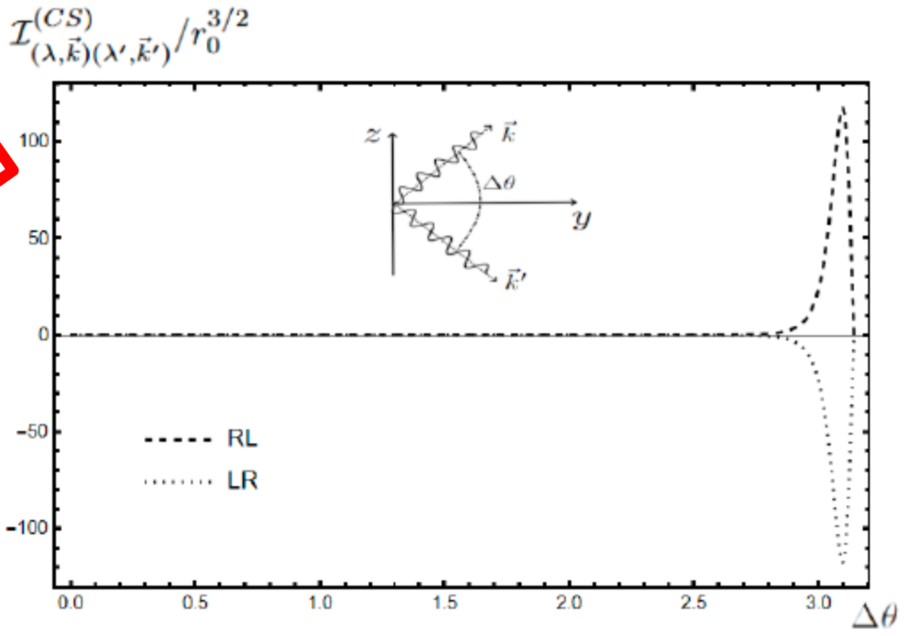

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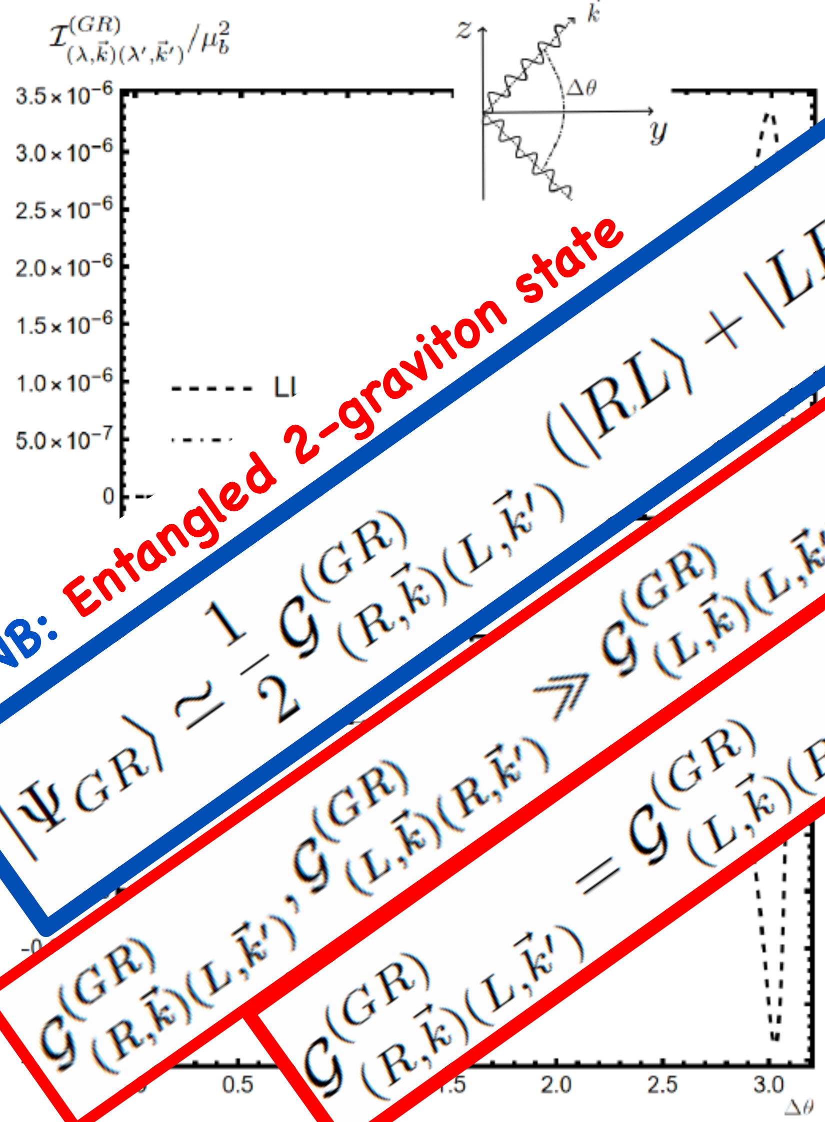

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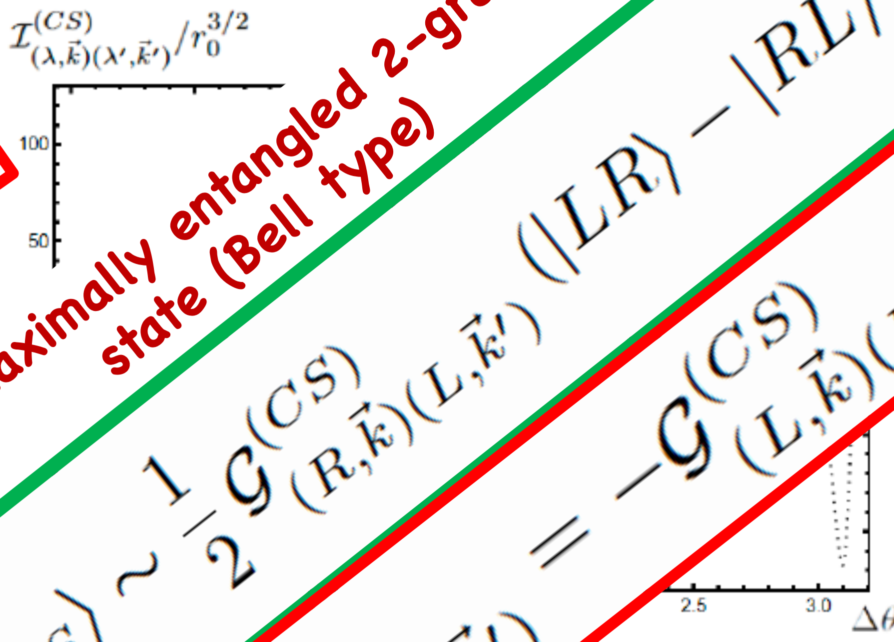

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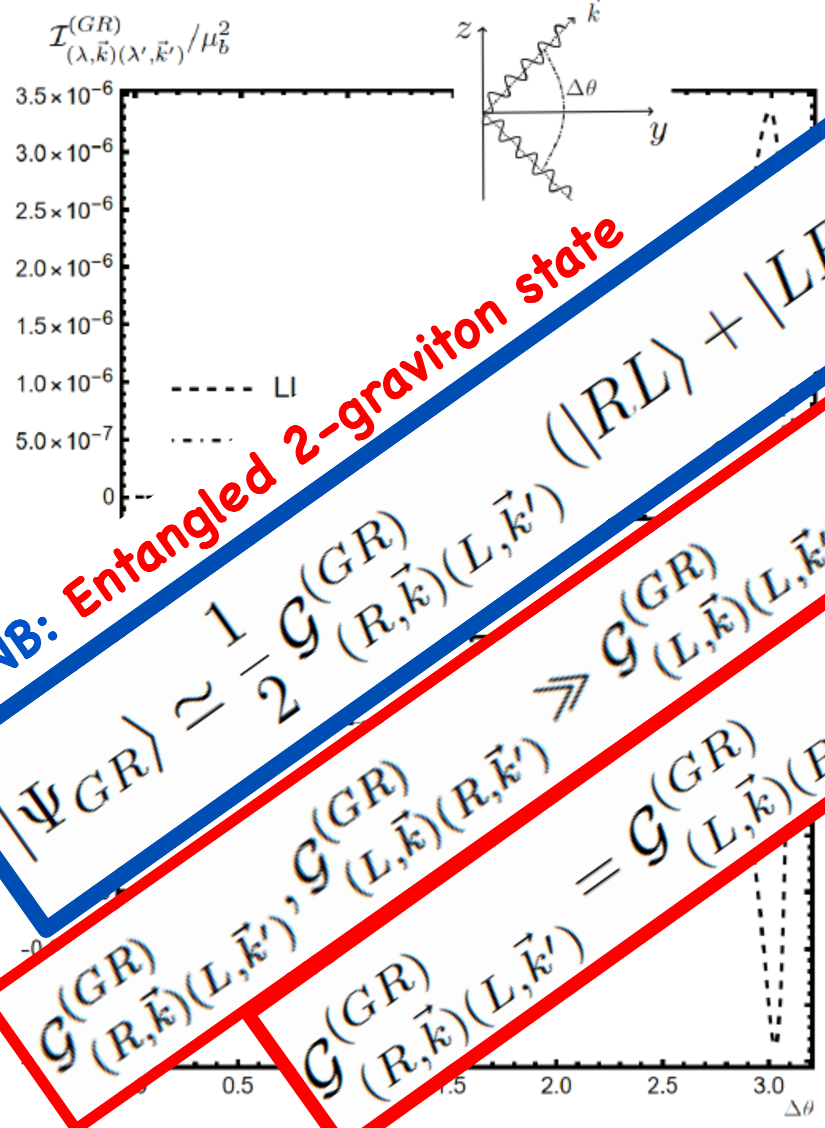
GR correlations in absence of anomalies



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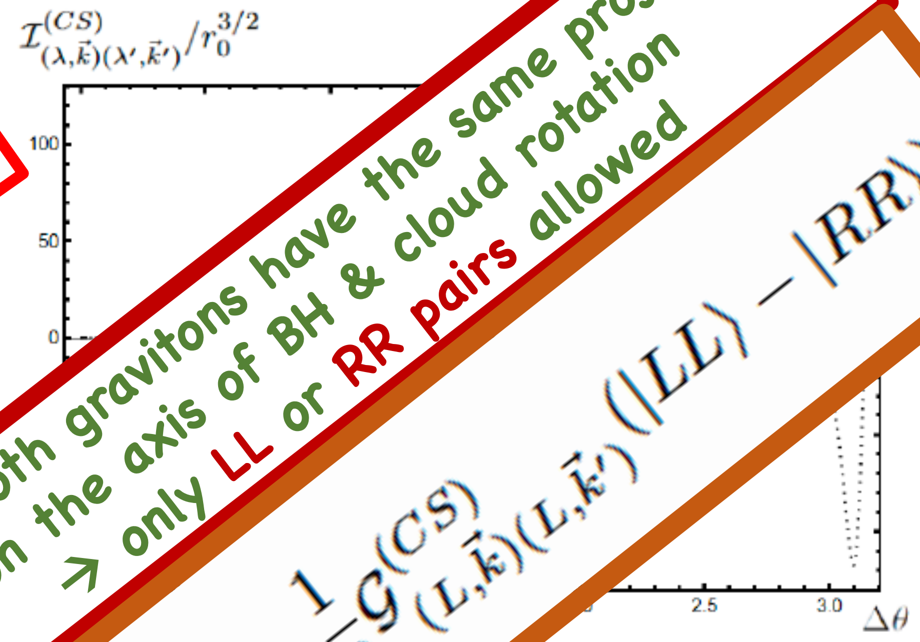


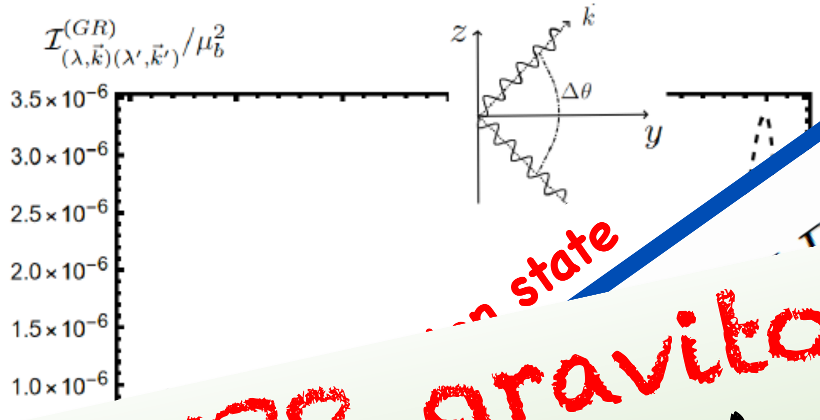
...ation correlations for the CS in- tera... pposite polarizations are produced; Maxi... occurs between the L and R polariza- tions. T... responds to $a_\mu = 0.1$.

NB: if both gravitons have the same projection on the axis of BH & cloud rotation → only LL or RR pairs allowed

$$|\Psi_{CS}\rangle = \frac{1}{2}\,\mathcal{G}^{(CS)}_{(L,\vec{k})(L,\vec{k}')}\left(|LL\rangle - |RR\rangle\right)$$

NB:

GR correlations in absence of anomalies



Contributions of CS gravitation...
Anomaly in EPR entan...

EPR graviton polarization entanglement depends on its origin:
GR-induced EPR correlations are ~ L ↔ R symmetric,
while CS anomalous terms induce L ↔ R antisymmetric

... polarization correlations for the GR ... The 2p-state results in the asymmetry between the LL and RR pairs. The plot corresponds to $a_\mu = 0.1$.

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Observational Prospects, current bounds of Graviton Squeezing & Axion Phenomenology

Above, we have seen that...

In CS gravity, squeezing effects (at least for realistic models) due to CS gravitational anomaly terms are subleading to those due to the GR terms of the effective action

GR squeezing:

$$\boxed{\sum_{I,J} |\mathcal{G}^{(GR)}_{IJ}|^2 \lesssim 2.5 \times 10^{-15}\, T \mu_b}$$

$$\boxed{\tau_s = \frac{1}{\omega_I(2p)} = 24\, \mu_b^{-1} \left(\frac{\alpha}{GM}\right)^{-1} a_\mu^{-8}}$$

$$\alpha/(GM) \sim \mathcal{O}(1)$$
$$a_\mu \equiv GM\mu_b = 0.1$$

Life time of cloud can be sufficiently long compared to the characteristic scale τ_s superradiance is effective,

eg $T > 10^7\, \tau_s$

D. Baumann, H. S. Chia, and R. A. Porto, PRD 99 (2019), 044001

→ significant multimode squeezing

$$\boxed{\mathcal{O}(0.1) \le \sum_{I,J} |\mathcal{G}^{(GR)}_{IJ}|^2 \lesssim \mathcal{O}(60-75)} \Rightarrow \langle N_{gr}\rangle \lesssim \mathcal{O}(10^6 - 10^7) \gg 1$$

if upper bound saturated

Graviton single-mode Squeezing can already be constrained from current LIGO-Virgo data

M.P. Hertzberg, J. Litterer, JCAP 03 (2023) 009

Assumption that a single mode k^* is significantly squeezed in the GW from merging of two BH (which LIGO-Virgo observe):

Gaussian profile of squeezed modes, with the peak corresponding to the characteristic frequency

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squeezing function

wavefunction

$$\psi_s(h,t) \propto \prod_{a=1,2} \prod_{\mathbf{k}} \prod_{p=+,\times} \exp\left[i\,\epsilon_{a\mathbf{k},p} + \frac{i}{2\hbar}\pi_{ac,\mathbf{k},p}\tilde{h}_{a\mathbf{k},p} - \frac{k\,S_{a\mathbf{k},p}(t)}{64\pi V G\hbar}\left(\tilde{h}_{a\mathbf{k},p} - \tilde{h}_{ac,\mathbf{k},p}(t)\right)^2\right]$$

like collection of harmonic oscillators

$$\tilde{h}_{ac,\mathbf{k},p}(t)$$ solution of classical graviton eqs $$\ddot{\tilde{h}}_{ac,\mathbf{k},p} = -k^2\tilde{h}_{ac,\mathbf{k},p}$$

$$\tilde{h}_{\mathbf{k},p}(t) = \int d^3x\, h_p(\mathbf{x},t)\, e^{-i\mathbf{k}\cdot\mathbf{x}}$$

(Fourier transform of GW perturbation)

$$S_{a\mathbf{k},p}(t) = \mathrm{Tanh}\left(\mathrm{Tanh}^{-1}(\beta_{a\mathbf{k},p}) + i\,k\,t\right)$$

$$S_{a\mathbf{k},p}(0) = \beta_{a\mathbf{k},p}$$

$$\epsilon_{a\mathbf{k},p}(t) = -\frac{k}{4}\int_0^t S_{a\mathbf{k},p}(\tau)d\tau - \frac{1}{4\hbar}\tilde{h}_{ac,\mathbf{k},p}(t)\pi_{ac,\mathbf{k},p}(t)$$

NB: **Squeezed states of Harmonic Oscillator wavefunction**

similar to coherent states, but product of variances does not saturate the uncertainty principle and it depends on time

$$\psi_s(x,0) \propto \exp\left[\frac{i}{\hbar}p_0(0)x - \frac{\beta}{2\hbar}m\omega_0(x-x_0(0))^2\right]$$

β is the "squeezing parameter". $\beta \neq 1$

$\beta = 1$ → retrieve the coherent state

Solution of time dependent Schroedinger 's equation yields:

$$\psi_s(x,t) \propto \exp\left[i\,\epsilon(t) + \frac{i}{\hbar}p_0(t)x - \frac{S(t)}{2\hbar}m\omega_0(x-x_0(t))^2\right]$$

$$\epsilon(t) = -\frac{1}{2}\omega_0\int_0^t dt'\,S(t') - \frac{1}{2\hbar}x_0(t)p_0(t)$$

Squeezing function

$$S(t) = \mathrm{Tanh}\left(\mathrm{Tanh}^{-1}(\beta) + i\,\omega_0\,t\right)$$

$$S(0) = \beta\,.$$

Graviton single-mode Squeezing can already be constrained from current LIGO-Virgo data

M.P. Hertzberg, J. Litterer, JCAP 03 (2023) 009

Assumption that a single mode k* is significantly squeezed in the GW from merging of two BH (which LIGO-Virgo observe):

Gaussian profile of squeezed modes, with the peak corresponding to the characteristic frequency

$$\boxed{\sigma_S = \sqrt{4\pi}\, e^{\zeta_p} \left(\frac{2\pi f^*}{\omega_{\rm Pl}} \right)}$$

$$f^* \sim 200\,\mathrm{Hz}$$

$$\omega_{\rm Pl} \equiv \frac{1}{\sqrt{G\hbar}} \approx 1.9 \times 10^{43}\,\mathrm{sec}^{-1}$$

data from the event GW150914

$$\boxed{\sigma_H \approx \sigma_L \approx 0.16 \times 10^{-21}}$$

The non- observation squeezing effects put constraints on squeezing parameters

$$\boxed{\zeta_p < 41}$$

Such an analysis should be extended to our entangled squeezed graviton states

If the above bound is valid then, in view of

$$\boxed{\sum_{I,J} |\mathcal{G}_{IJ}^{(GR)}|^2 \lesssim 2.5 \times 10^{-15}\, T \mu_b\,.}$$

ζ^2 **the square of the squeezing parameter in the single-mode analysis**

Hence **if $\zeta < 41$** (if previous analysis applicable to our case) → the long-life time ***T*** **=$10^7\,\tau_s$** situation for axionic clouds is **compatible** with **current interferometer data** (LIGO-Virgo etc)

In general the (**saturated**) **upper bounds** on the **squeezing parameter,** will impose **upper bounds** on the **allowed** **axion-cloud life time**

$$T < 6.7 \times 10^{17}\, \mu_b^{-1}$$

Such an analysis should be extended to our entangled squeezed graviton states

If the above bound is valid then, in view of

$$\boxed{\sum_{I,J} |\mathcal{G}_{IJ}^{(GR)}|^2 \lesssim 2.5 \times 10^{-15}\, T \mu_b\,.}$$

$T = 10^7\, \tau_s$

D. Baumann, H. S. Chia, and R. A. Porto, PRD 99 (2019), 044001

ζ^2 **the square of th**
parameter in the
analys

For $\mu_b\, \tau_s$ **= 2.4 x 10^9**
we obtain
T < 2.8 x 10^8 τ_s
for the **allowed** range

Hence **if ζ < 41** (if previous analysis applicable to our c
the long-life time $T = 10^7\, \tau_s$ situation for axionic clouds
is **compatible** with **current interferometer data** (LIGO-Virgo etc)
In general the (**saturated**) **upper bounds** on the **squeezing parameter,**
will impose **upper bounds** on the **allowed** **axion-cloud life time**

$$\boxed{T < 6.7 \times 10^{17}\, \mu_b^{-1}}$$

Future Detection

Future detection of such gravitons using combinations of interferometers could justify, or falsify, depending on the strength of the signal, the assumption on the EFT nature of QG.

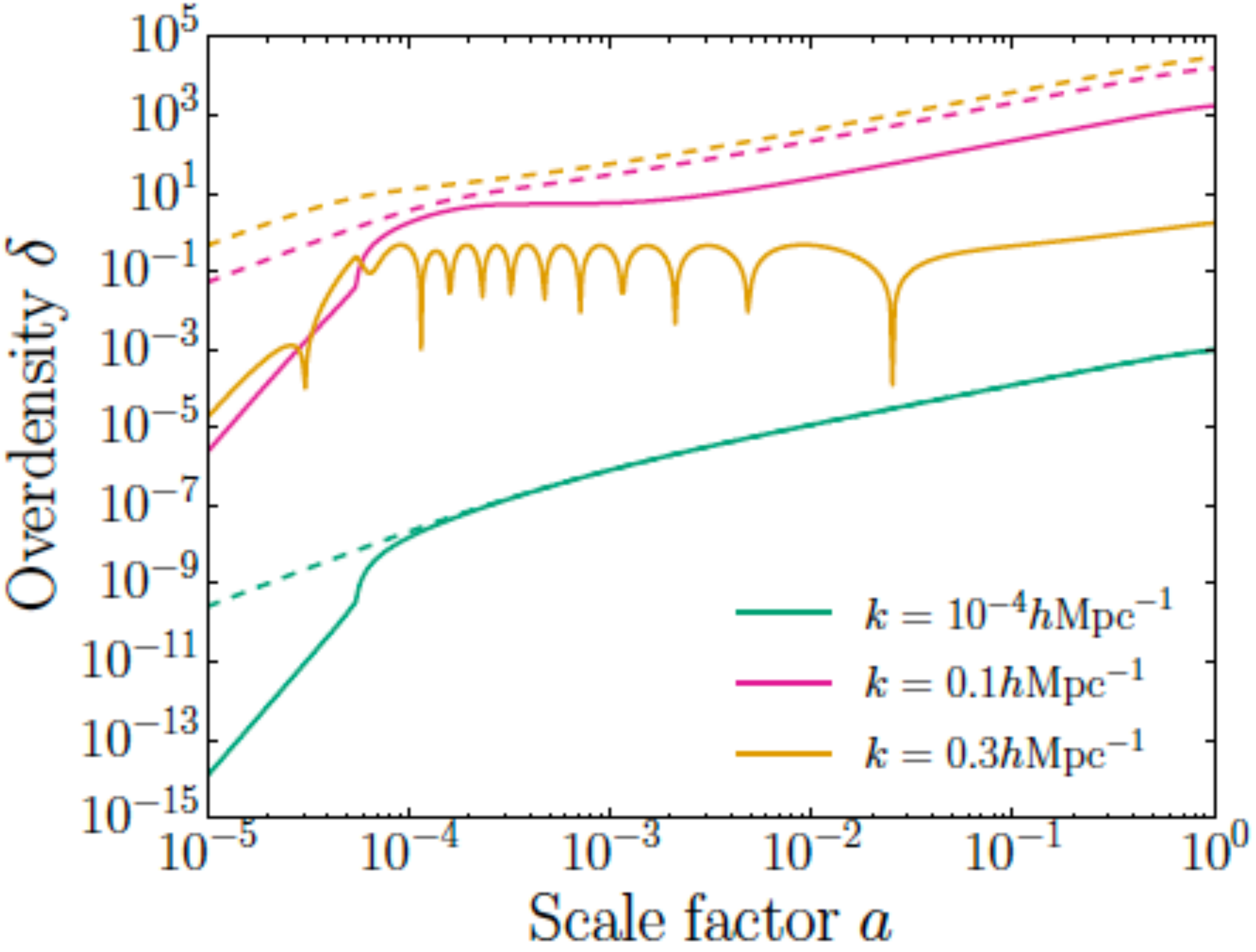
Observation of Polarization entangled gravitons is direction dependent
Tracing out unobserved d.o.f. creates additional ``thermal" states, beyond those by conventional sources, → indirect clue for detection in future facilities, quantum tech. ?

AXION PHENOMENOLOGY

Ultra Light Axion (ULA) DM (allowed in string theory)

Contribution to galactic growth if dominant DM species

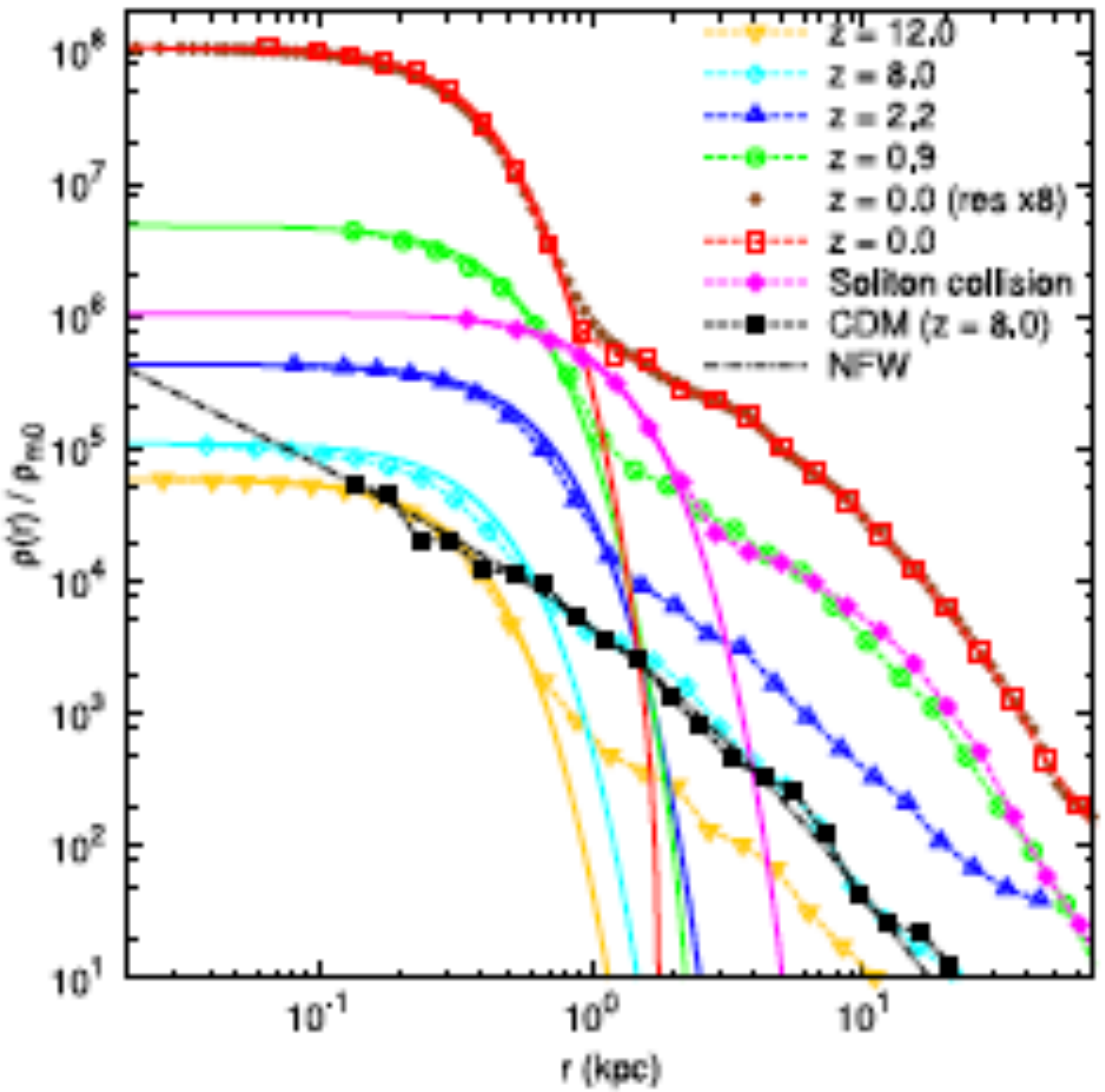
D.J.E. Marsh,
Phys. Rept. 643, 1 (2016)
[arXiv:1510.07633 [astro-ph.CO]].



$$m_a = 10^{-26}\ \mathrm{eV}$$

R. Hlozek, D. Grin, D. J. E. Marsh, and P. G. Ferreira, Phys. Rev. D**91**, 103512 (2015), 1410.2896.

Halo Density Profiles and ULA



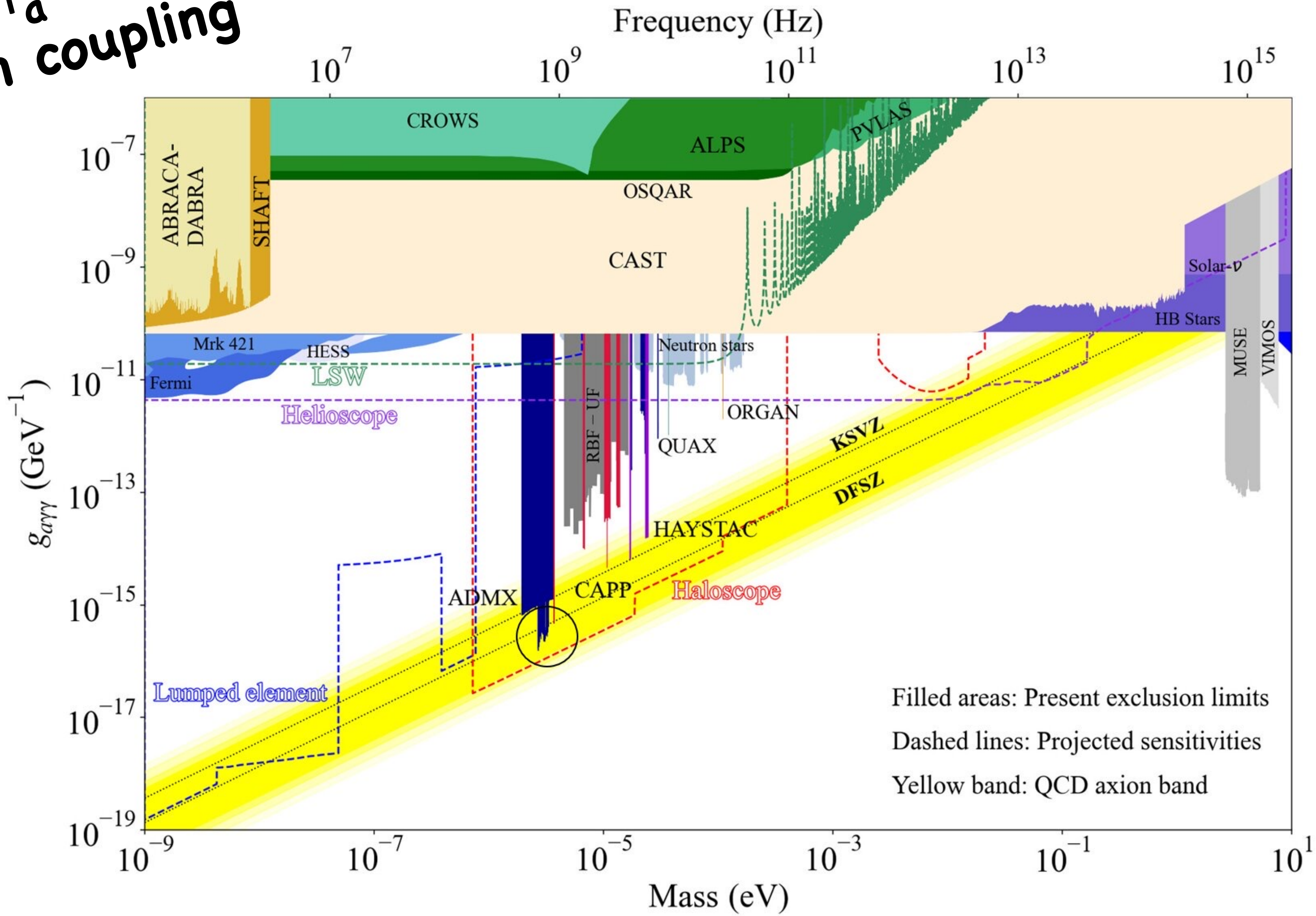
$$m_a = 8.1 \times 10^{-23}\ \mathrm{eV}$$

D.J.E. Marsh,
Phys. Rept. 643, 1 (2016)
[arXiv:1510.07633 [astro-ph.CO]].

Current Axion Searches

http://www.ibs.re.kr/en/
Credit Inst. for Basic Science

$g_{s\gamma\gamma} = f_a^{-1}$
axion coupling



BBN Constraints on f_a

$$\mathcal{L}_{\rm int} = -\frac{g_{\phi\gamma}}{4}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{g_{\phi N}}{2m_N}\partial_\mu\phi(\bar{N}\gamma^\mu\gamma_5 N) + \frac{g_{\phi e}}{2m_e}\partial_\mu\phi(\bar{e}\gamma^\mu\gamma_5 e) - \frac{i}{2}g_d\phi\bar{N}\sigma_{\mu\nu}\gamma_5 N F^{\mu\nu} ,$$

Nucleon
or in general massive
fermion ψ of mass m_f

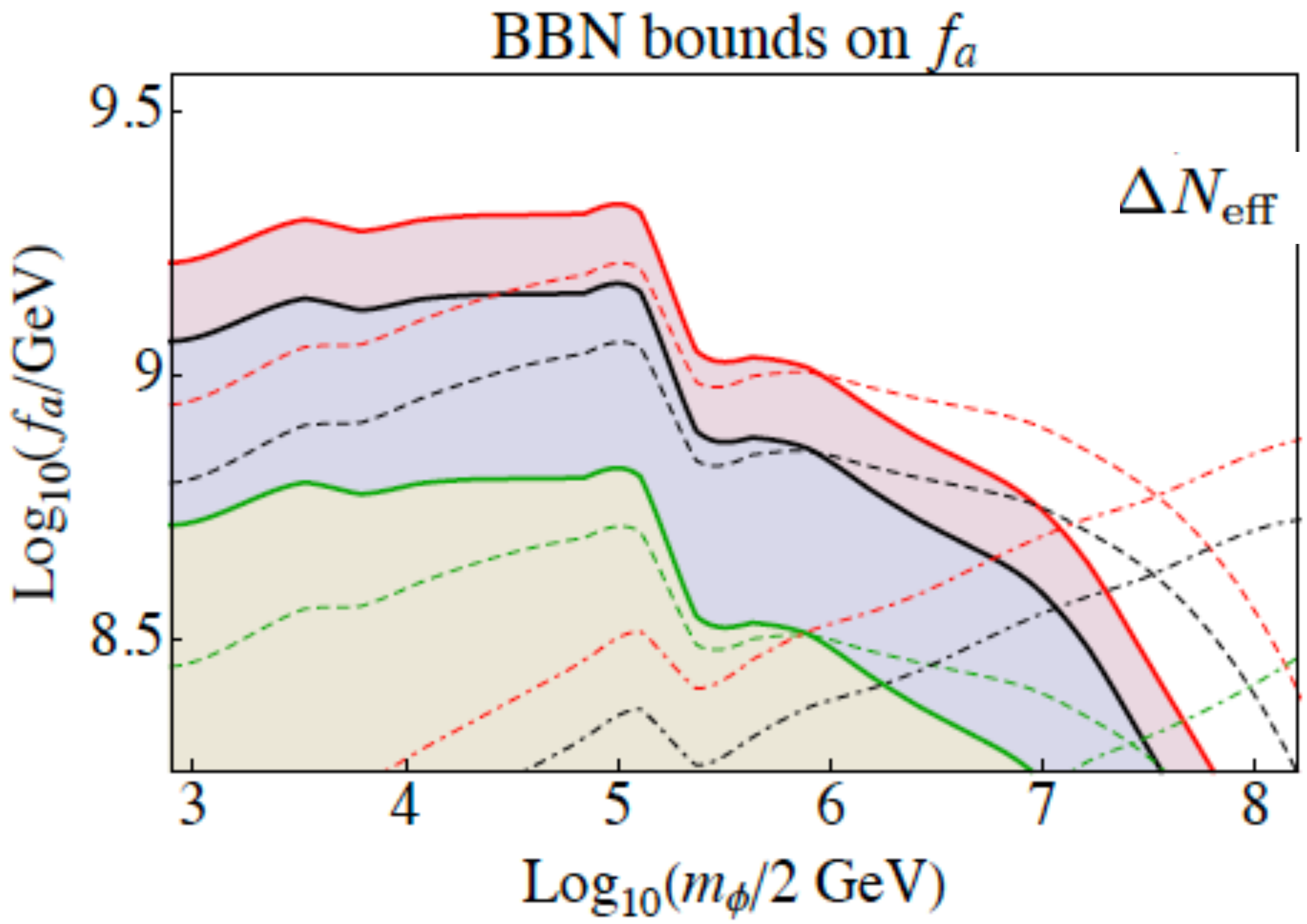
$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$$

ϕ = axion, Use massive fermion equations of motion so as t
to obtain effective axion-fermion interactions :

$$\mathcal{L}_f = c_f m_f \phi\bar{\psi}\gamma^5\psi/f_a$$

Implying production of heavy fermions *f*, via $a + \gamma \rightarrow f + \bar{f}$
which can alter the proton to neutron ratio during BBN

$c_f = 1 \quad \mathcal{L}_f = c_f m_f \phi \bar{\psi} \gamma^5 \psi / f_a$



$\Delta N_{\rm eff} = 0.1, 0.5, 1$ (green, black, red)

J. P. Conlon and M. C. D. Marsh, JHEP**10**, 214 (2013), 1304.1804.

BBN constraints rule out $f_a \leq 10^9$ GeV for a wide range of Masses m_ϕ.

D.J.E. Marsh,
Phys. Rept. 643, 1 (2016)
[arXiv:1510.07633 [astro-ph.CO]].